

## Levels of Sophistication in Student Reasoning

### Place Value<sup>1</sup>

Level	Sublevel	Description
0		<b>Student has difficulties counting by ones.</b>
1		<b>Student operates on numbers as collections of ones (no skip-counting by place value).</b>
	1.1	Student correctly counts groups of objects by ones but cannot count groups of ten.
	1.2	Student correctly counts groups of ten.
	1.3	Student operates on tens and ones separately as ones.
2		<b>Student operates on numbers by skip-counting by place value (e.g., counts by tens).</b>
	2.1	Student counts by tens and ones separately.
	2.2	Student counts by tens in mid-decades.
3		<b>Student operates on numbers by combining and separating place-value parts (e.g., adds tens parts without counting).</b>
	3.1	Student uses multiples-of-ten language ("forty plus twenty equals sixty").
	3.2	Student uses tens language ("4 tens plus 2 tens equals 6 tens").
	3.3	Student integrates Levels 2.1, 2.2, 3.1, and 3.2.
4		<b>Student understands place value in expanded algorithms.</b>
	4.1	Student understands place value in expanded addition and subtraction algorithms.
	4.2	Student understands place value in expanded multiplication and division algorithms.
5		<b>Student understands place value in traditional algorithms.</b>
	5.1	Student understands place value in traditional addition and subtraction algorithms.
	5.2	Student understands place value in traditional multiplication and division algorithms.
6		<b>Student generalizes place-value understanding to larger numbers, numbers less than 1, and exponential notation.</b>

### Addition & Subtraction<sup>2</sup>

Level	Sublevel	Description
0		<b>Student does not understand addition and subtraction situations.</b>
1		<b>Student adds or subtracts numbers as collections of ones (no skip-counting by place value).</b>
	1.1	Student counts all.
	1.2	Student counts on or down.
	1.3	Student recalls or derives a fact.
	1.4	Student operates on tens and ones separately as ones.
2		<b>Student adds or subtracts numbers by skip-counting place-value parts.</b>
	2.1	Student counts by tens and ones separately.
	2.2	Student counts by tens in mid-decades.
3		<b>Student adds or subtracts numbers by combining or separating place-value parts.</b>
	3.1	Student uses tens landmarks.
	3.2	Student separately adds or subtracts the tens and ones parts of one number to or from the other number.
	3.3	Student combines or separates tens parts and ones parts and then adds the results.
4		<b>Student uses and understands expanded addition and subtraction algorithms.</b>
5		<b>Student uses and understands traditional addition and subtraction algorithms.</b>

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<sup>1</sup>Battista, M. T. (2012). *Cognitive-Based Assessment & Teaching of Place Value: Building on Students' Reasoning*. Heinemann.

<sup>2</sup>Battista, M. T. (2012). *Cognitive-Based Assessment & Teaching of Addition and Subtraction: Building on Students' Reasoning*. Heinemann.