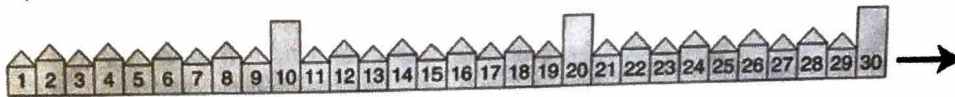


Mental Math and Algorithms

Task:

- If I'm at building 17 and I go forward 5 buildings, where am I?
- If I'm at building 17 and I go forward 10 buildings, where am I?
- If I'm at building 17 and I go forward 20 buildings, where am I?



AS 1.2

Response:

- Student counted on 5 buildings from 17 and said 22.
- Student started at 17 and counted on 10 buildings to 27.
- Student started at 17 and counted 1-13 on buildings 18-30. He then counted 14-18 on the 5 fingers of his left hand and 19-20 on his right hand. Because there were 2 fingers showing on this right hand, he answered 32.

This student correctly counted on when all the buildings he needed to count were visible but could not count on when some of the needed buildings were not visible.

Task: $535 - 342 =$

Response 1: $5 - 2 = 3$. You can't take 40 from 30, so I have to take 100 from the 500 and add it to the 30. [Rewriting the problem as shown below] That makes the 30 into 130 and the 500 into 400. So, $130 - 40 = 90$; $400 - 300 = 100$. $100 + 90 + 3 = 193$.

$\begin{array}{r} 500 + 30 + 5 \longrightarrow \\ - (300 + 40 + 2) \\ \hline 3 \end{array}$	<p><i>Rewriting</i></p> $\begin{array}{r} 400 + 130 + 5 \\ - (300 + 40 + 2) \\ \hline 100 + 90 + 3 = 193 \end{array}$
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Response 2: [Writing as shown below] Five hundred minus three hundred equals two hundred. Thirty minus forty equals minus ten. Five minus two equals three. Two hundred minus ten is one hundred ninety, plus three, equals one hundred ninety three.

$$\begin{array}{r} 535 \\ -342 \\ \hline 200 \quad 500 - 300 = 200 \\ -10 \quad 30 - 40 = -10 \\ \hline 3 \quad 5 - 2 = 3 \\ \hline 193 \end{array}$$

PV 4.1

AS 1.3

Task: $9 + 7 =$

Response: 16. It's in our mathematics book all the time.

PV 1.2

Task: How many cubes are in 3 stacks of ten?

Response: The student counted by ones, putting up one finger when she reached ten, a second finger when she reached twenty, and a third finger when she reached thirty.

This student knew that as she was counting by ones, each of the numbers 10, 20, and 30 represented an additional group of ten ones.

AS 4

Task: $(200 + 30 + 4) + (300 + 40 + 2) =$

Response: [Student rewrites the problem as $200 + 300 + 30 + 40 + 4 + 2$. She draws segments joining the hundred's addends, segments joining the ten's addends, and segments joining the one's addends.]



$200 + 300 = 500$. $30 + 40 = 70$. $4 + 2 = 6$. [Joining 500 and 70 with segments] $500 + 70 = 570$. [Joining 570 and 6 with segments] plus 6 = 576.

PV 2.1

Task: Mr. Smith has 35 candies in one hand and 24 candies in his other hand. How many candies does he have altogether?

Response: [Using fingers] 10, 20, 30, 40, 50; 55, 56, 57, 58, 59. It's 59.

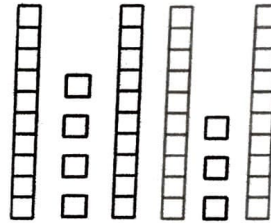
PV 3.1

Task: In a box, there are 35 red apples and 27 green apples. How many apples are in the box?

Response 1: 30 plus 20 is 50. And then add the 7 and 5, equals 12. So 50 and 12 is 62.

Response 2: 35 plus 20 is 55; plus 5 is 60, plus 2 is 62.

Task: How many squares are there altogether?



PV 1.3

Response: [Pointing at the first ten-strip from the left] 10 [Pointing at the next 4 individual squares] 11, 12, 13, 14. [Pointing at the next two ten-strips] 15, 16. [Pointing at the next 3 individual squares] 17, 18, 19. [Pointing at the last ten-strip] 20.

This student lost track of the values of the ten-strips.

AS 5.2

Task: George has 24 dollars. How many more dollars does he need so that he has 47 dollars?

Response: Add 20 to 24 to get 44 then add 3 to 44 to get 47. So, 20 plus 3 equals 23.

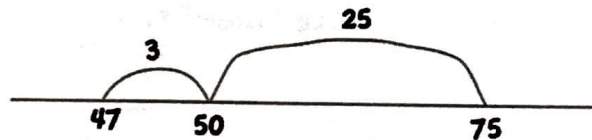
AS 2.1

Task: Emily has 54 teddy bears. She gives 26 of her teddy bears to Liz. How many teddy bears does Emily have left?

Response: Well, 54 - 20 is 34. Then minus that 6; that'd be 28.

Task: $75 - 28 =$

Response: [Going right to left on an empty number line as shown below] 75 minus 25 is 50. Then minus 3; that equals 47.



AS 3.1

This student used the 50 landmark in separating parts of 28 to subtract from 75.

Task: $45 \times 34 =$

Response: 40 times 30 is—4 times 3 is 12 add the two zeros—one-thousand two-hundred. 30 times 5 is 150 because 3 times 5 is 15 and add the zero, 150. 40 times 4 is 160 because 4 times 4 is 16 and just add the zero. 4 times 5 is 20. Add 1200, 150, 160, and 20 and you get 1,540. [Student made an addition error.]

PV 4.2

$$\begin{array}{r}
 30 \times 40 = 1200 \\
 30 \times 5 = 150 \\
 40 \times 4 = 160 \\
 4 \times 5 = 20 \\
 \hline
 1540
 \end{array}$$

Task: Mary has 84 cookies. She wants to divide them equally among four people. How many cookies does each person get?

Response: I think you are going to have to divide 84 by 4. Four times 3 is 12 [recording 3 in the right of the problem]. 72 [subtracting 12 from 84]. 10 groups [recording 10 under the 3]. 40. 32 [subtracting 40 from 72]. 4 times 5 is 20. 12 [subtracting 20 from 32]. And 3 times 4 is 12 again. 12 and 12 cancel each other out; zero, zero. So you would add these together: 10 plus 5 is 15, plus 3 is 18, then another 3 is 21. So each person would get 21 cookies.

$$\begin{array}{r}
 21 \\
 4 \overline{)84} \\
 \underline{-12} \\
 72 \\
 \underline{-40} \\
 32 \\
 \underline{-20} \\
 12 \\
 \underline{-12} \\
 0 \\
 \hline
 0
 \end{array}$$

Task: $46 =$ _____ tens and _____ ones.

Response: 46 equals 40 plus 6. [Writes the following:]

$46 = \underline{40}$ tens and $\underline{6}$ ones

PV 1.1

This student separated 46 into its multiples-of-ten and ones parts, but she did not understand the multiples-of-ten part in terms of groups of ten.