

Consider four important elements of setting up challenging mathematics problems to support all students' learning. Kara J. Jackson, Emily C. Shahan, Lynsey K. Gibbons, and Paul A. Cobb

Mathematics lessons can take a variety of formats. In this article, we discuss lessons organized around complex mathematical tasks. These lessons usually unfold in three phases (Van de Walle et al. 2010). First, the task is introduced to students. Second, students work on solving the task. Third, the teacher "orchestrates" a concluding whole-class discussion in which students are supported to make mathematical connections between solutions and to develop conceptual understanding of significant mathematical ideas (Smith et al. 2009).

Complex tasks invite students to generate multiple solutions and justify

their reasoning (Stein et al. 2000). Such tasks live up to their potential only if students engage productively in the task. Therefore, the first phase, the task introduction, is crucial. We describe how teachers can set up, or "launch" (Lappan et al. 2009), complex tasks by leading introductory wholeclass discussions in which they help students engage in the task and learn significant math.

A task's setup impacts both what students *and* the teacher are able to achieve during a lesson. When a complex task is launched well and students are engaged, the many solutions generated can form the basis of the concluding whole-class discussion. Ideally, while students are solving the task, the teacher circulates around the room to plan for that concluding discussion. Students are more likely to learn from a summary discussion if they have been able to engage productively in the task.

Our work with middle-grades mathematics teachers and our own classroom experiences indicate that students frequently struggle when attempting to solve complex tasks because they do not understand various aspects well enough to get started (Jackson et al. 2011). Teachers, in turn, often respond to students'

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struggles by repeating the task to individuals or groups of students. This takes up a significant amount of time and can adversely affect the remainder of the lesson.

When teachers are spending time relaunching a lesson, they are often unaware of how other students are solving the task. Therefore, they cannot plan well for the concluding whole-class discussion. In addition, to keep the lesson moving, teachers often suggest particular solution methods to get students started. Such explicit suggestions usually cause students to solve the task in the same way, which lessens the chance that a rich concluding whole-class discussion will occur. The end result is a diminishing of students' opportunities to learn significant mathematics.

How can a teacher launch a complex task so that students are able to engage productively in solving it? We analyzed videorecordings of lessons conducted by 132 middle-grades mathematics teachers who used Standards-based curricula to determine important aspects of effective launches (Jackson et al. 2011). We identified four aspects of launches that enabled the majority of students to engage in solving a complex task and to participate in a concluding whole-class discussion that focused on conceptual understanding. We illustrate these aspects by describing how a seventh-grade teacher, Smith (all names are pseudonyms), effectively introduced a complex task, Dollars for Dancing (see fig. 1), in his classroom.

Dollars for Dancing is a *Standards*based task from a unit on linear relationships that emphasizes connections among tables, graphs, and equations. Smith's seventh graders had used tables, graphs, and equations in prior lessons to solve problems involving linear relationships with *y*-intercepts of zero. This lesson was their first encounter with a linear relationship and nonzero *y*-intercepts. Smith's goal in using the Dollars for Dancing task was to support his students' understanding of the *y*-intercept as an initial value and slope as a constant rate of change.

Smith devoted eight minutes of an hourlong lesson to launching this task. All his students were then able to engage productively in finding a solution, and most contributed to the concluding whole-class discussion in which they made connections among various strategies and representations. In doing so, they explained their reasoning in ways that indicated conceptual understanding of key features of linear relationships (Jackson et al. 2011).

# **CRITICAL ELEMENTS**

We list four crucial aspects to keep in mind when setting up complex tasks to support all students' learning.

# 1. Discuss the Key Contextual Features

Complex tasks often involve real-world scenarios that have been chosen to support the development of students' reasoning and communication about particular mathematical ideas (Hiebert et al. 1997). Some students may struggle when starting because they are unfamiliar with the context, or scenario, in which the task is grounded. It is therefore important that the teacher and students discuss any potentially unfamiliar features of the task. For

**Fig. 1** The Dollars for Dancing task allowed Smith's seventh graders to experience a linear equation with a nonzero *y*-intercept.

# **DOLLARS FOR DANCING**

Three students at a school are raising dollars for the school's Valentine's Day Dance. All three decide to raise their money by having a dance marathon in the cafeteria the week before the real dance. They will collect pledges for the number of hours that they dance, and then they will give the money to the student council to get a good DJ for the Valentine's Day Dance.

- Rosalba's plan is to ask teachers to pledge \$3.00 per hour that she dances.
- Nathan's plan is to ask teachers to give \$5.00 plus \$1.00 for every hour he dances.
- James's plan is to ask teachers to give \$8.00 plus \$0.50 for every hour he dances.

**Part A.** Create at least three different ways to show how to compare the amounts of money that students can earn from their plans if they each get one teacher to pledge.

**Part B.** Explain how the hourly pledge amount is represented in each of your ways from part A.

**Part C.** For each of your ways in part A, explain how the fixed amount in Nathan's plan and in James's plan is represented.

**Part D.** For each of the ways in part A, show how you could find the amount of money collected by each student if they could dance for 24 hours.

Part E. Who has the best plan? Justify your answer.

Source: Adapted from Task 1.3 ("Raising Money"), Connected Mathematics Project 2 grade 7 book, Moving Straight Ahead: Linear Relationships (Lappan et al. 2009)

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example, key contextual features of Dollars for Dancing include understanding that people organize dance marathons to raise funds and that dance marathons last a long time. Unless these elements are discussed, students who are unfamiliar with a dance marathon might struggle early.

Smith anticipated that some of his students were indeed unfamiliar with the dance-marathon scenario. He began his launch by eliciting students' prior knowledge about dance marathons by projecting Internet images and asking students to discuss what they saw. Students replied, "Dance" and "Dance marathon." He built on students' contributions to develop a common way to describe dance marathons as "groups of people who dance for a certain amount of time." He then pressed students to explain why people might hold such an event. Their proposals were the foundation to explain that the task was a fundraiser to pay for a DJ for the school's Valentine's Day Dance.

How can teachers help students understand the key contextual features of a task? One idea is to ask students to imagine that they are participants in the scenario and to share what they know about it. Another idea involves projecting images relevant to the scenario and asking students what they know about it. Images coupled with discussion are especially useful strategies to support students who are English language learners (ELLs) (Moschkovich 1999).

Teachers could also connect the task scenario to a person, place, or event that might be familiar or of interest to students. For example, Smith personalized the Dollars for Dancing task for students by connecting it to a school activity (the upcoming Valentine's Day Dance) to generate student interest and to give students additional ways to contribute to the discussion (McDuffie et al. 2011). In the effective launches that we identified, teachers did not simply talk to students about the key features of tasks but instead solicited input from their students.

### 2. Discuss the Key Mathematical Ideas

Our analysis of instruction indicated that focusing solely on context is not enough. It is also critical to discuss the key mathematical ideas without hinting at particular methods or procedures that should be used to solve the task. For example, students were expected to use tables, graphs, and equations in Dollars for Dancing to represent the accumulation of money over time in the three different fund-raising plans. To meet these expectations, students needed to understand several ideas:

- 1. Money accumulates as a participant continues to dance for a greater number of hours.
- Different ways exist to accumulate money, such as starting with a fixed amount or earning a fixed amount of money per hour of dancing.

Unless students understood this distinction, they would be unlikely to

make connections among the different ways of accumulating money, the slope, and the *y*-intercept. It is also probable that some students would struggle both to create appropriate tables, graphs, and equations, and later to comprehend their peers' representations and explanations during the whole-class discussion.

Smith supported students' understanding of key mathematical ideas in the Dance Marathon task by explicitly discussing the difference between an up-front amount and an hourly amount.

There are two ways that you can raise money in a dance marathon. One way is to dance for a long time. If I give you \$0.50 every hour, you're going to make a lot of money. But there's another way that you could raise money, and that is to ask for a pledge. Not per hour, but just a [onetime] donation. We call that a donation. And you might go up to your teacher and say, "Can you give me \$6.00 for being in the dance marathon?" That's different. Can anybody explain how that is different if I say, "Can you give me \$6.00?" or instead "Can you give me \$0.50 an hour?"

Smith called on students to explain the distinction. Jasmine responded, "Either they pay you up front or ... they continue to pay you for however long you dance." As students shared their ideas, Smith asked students to restate what others said (e.g., "Can you say what Jasmine said in your own words?"). He also praised students' ideas and adopted students' ways of describing the distinction. Marisa offered, "One of them, you already start with it, and the other one, you have to kind of work for it to get more." Smith responded, "Exactly. I like the way that's worded. One of them you start with; you just have it. The other one, you have to work for it to get the money."

After the majority of students

could describe in their own words the distinction between the two ways to accumulate money, Smith handed out the task sheet and briefly explained students' responsibilities when working in their small groups. Students then began the task.

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To develop his students' understanding of key mathematical ideas, Smith focused students' attention on the distinction between fund-raising strategies, adopted students' language, and asked multiple students to state the distinction in their own words. We have also witnessed teachers asking students to act out aspects of the task to help develop an understanding of key mathematical ideas. Smith might have asked two students to pretend to be Rosalba and Nathan. Then he might have asked "Rosalba" (hourly amount) and "Nathan" (upfront amount) to explain how each earned money. Other students would then have been asked to explain the key distinction between the two plans (Rosalba had to earn all of her money, whereas Nathan received some money upfront).

# 3. Develop Common Language to Describe the Key Features

In the effective launches that we identified, teachers did not simply talk *to* students about the key features of tasks but instead solicited input *from* students. These teachers asked questions that required more than yes or no responses, which helped the teacher determine the level of support that students needed to engage in the task (Boaler 2002).

It was also critical for teachers to build on student contributions and both support and press students to develop common language to describe the key features of the task—contextual features, mathematical ideas, and any other language—that might be unfamiliar or confusing. For example, Smith anticipated that the first word in part A, "create," might be troublesome for his students, especially ELLs (see **fig. 1**). He therefore asked students to explain the meaning of "create" using their own words during the setup.

Why is *developing common language* so important? Developing common language gives students a way to communicate with one another while working in small groups and participating in the whole-class discussion. Teachers can use strategies similar to those described by Smith to support the development of common language, such as highlighting particular ideas, adopting students' language, asking students to describe key aspects in their own words, and asking students to restate what others have said (Chapin, O'Connor, and Anderson 2003).

## 4. Maintain the Cognitive Demand

To maintain the cognitive demand, or mathematical rigor, of the task (Stein et al. 2000), avoid suggesting a particular solution method to students. Doing so robs them of the opportunity to develop mathematical understanding as they generate their own solution methods and representations. Moreover, if students solve the task in the same way, it is unlikely that the concluding whole-class discussion will present students with further opportunities to develop conceptual understanding of mathematical ideas.

With Dollars for Dancing, Smith could have assisted his students by constructing a table, graph, or equation. Although this would help all students get started, it would have also reduced the cognitive demand of the task. Instead, Smith maintained the rigor by helping students understand important aspects of the task while leaving solution pathways open. This action allowed students to reason about significant mathematical ideas both while solving the task and when discussing it at the conclusion.

# PLANNING COMPLEX TASKS

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Conducting high-quality launches requires considerable planning. **Figure 2** provides a set of questions that teachers can ask themselves when planning an effective launch.

Smith's launch was effective because he had identified clear learning goals for the particular lesson in light of the mathematics standards of the state in which he taught. He also anticipated contextual features, mathematical ideas, and language central to the task that might not be self-evident to his students. Clearly, time is of the essence in classroom instruction. Therefore, teachers have to make judgments about what warrants attention in the launch of a task. These judgments must be based on a clear set of mathematical goals for instruction and knowledge of what might be unfamiliar to students. However, the time spent planning for an effective launch is worth it. Students are much more likely to be able to get started solving a complex task, thereby enabling the teacher to attend to students' thinking and plan for a concluding whole-class discussion. This, in turn, increases the chances that all students will be supported to learn significant mathematics as they solve and discuss the task.

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# Fig. 2 These planning questions can help launch a complex task effectively.

## Mathematical Goals of the Lesson

- · What are the mathematical goals for this lesson?
- · On what prior mathematical understanding and skills does this task build?
- What is the new mathematics developed by this task?

#### Key Contextual Features of the Task

- · What are the key contextual features of the task?
- · Which features are likely to be unfamiliar to some or all of my students?
- · How will I elicit and develop students' understanding of these features?

#### Key Mathematical Ideas of the Task

- What key mathematical ideas do my students need to understand so that they will be able to engage in solving the task?
- Which ideas are likely to be unfamiliar to some or all of my students?
- How will I elicit and develop students' understanding of these ideas?

#### Development of Common Language

- Which additional language in the task statement is likely to be confusing or unfamiliar to some or all of my students?
- How will I support my students to develop common language to describe the key contextual features, mathematical ideas, and additional language central to the task?

#### Maintaining the Cognitive Demand

 What specifically do I need to avoid doing in the launch so that I maintain the cognitive demand of the task?

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