1 Normal distributions – Statistical models

Objective: To explain and illustrate how normal curves can be used in an everyday setting. We also illustrate situations that are not normally distributed.

Hello, and welcome to Math Matters, where we take some of the entry-level math and try to show you where it is useful in the everyday world. I’ve got some nice, neat stuff to show you today, so let’s get right after it. I want to talk to you first about normal distributions. I want to be honest with you – this is a topic we cover in our general math courses and I think we do a pretty good job showing applications for the normal distributions, and showing how you can use this as a predictive statistic. There are certain things that are impossible to measure while you are using the item, but since that has already been tested, you can infer its previous behavior from its tested behavior. And we use the normal distribution a lot of times to do that. They’re very useful.

The idea is that you have this nice symmetric curve. And it’s a good kind of representative statistic. And you look at things that are symmetric about an arithmetic mean, has this nice kind of decay on both sides. Some examples of that would be like most mechanical things. Error measurements from mechanically processes, lifespans of consumable items like light bulbs or lighters or something like that. And they also come into play when we talk about standardized test scores. Although that is not always true. We use normalized, I mean, normal distributions to describe test scores but not because it’s a test. It’s because of the way the test is designed. I am going to talk about that just a little later. (Figure 1)

The nice thing about the normal distribution is we calculate the standard deviation and that is our measuring stick. So if we go out, no matter what the mean is, no matter what the standard deviation is, if we go one standard deviation above and below the mean we always get the same percentage of data. There is roughly 68.3%. And if we go two standard deviations above and below the mean, we each the same percent, about 95\%\% . And if we go 3 standard deviations, you pretty much got the whole shooting match. There is a little bit of outlier stuff out past that but not very much. (Figure 1)

Really nothing is exactly normally distributed. What, what, where this comes about is you start taking data and you start tallying the data and counting the frequency. And you get kind of this nice bell shaped curve. So here are some IQ tests, for example, that have that are designing to have a mean of 100 standard deviations of 15. And what you are looking at here is me tallying up – I did this on the computer obviously – tallying up all the different scores between 55 and 145 or so. And you can see they are rising and you can see the general shape that it is taking. It’s getting that nice kind of normal curve. And I will let it run a couple more minutes here and then I will show you that that curve is a good approximation to the actual data. Just let it, a little more, a little more. And about right, and about right there I’m staying, There! There you go. That blue curve, the data varies from a little bit,
but it is a good approximation. (Figure 2)

Okay, so here is how we use these things in everyday life. Say you have this light bulb and it is suppose to last 2,100 hours. And you know from testing, it has a standard deviation of a 100 hours. So when you start, when you take a light bulb out of the package you might want to say “Well, how long do I expect this light bulb to last?” 2100 hours. But what would be the probability that this particular bulb would last more than 1950 hours? I don’t know why you would want to know that, but say you wanted to know that. Well, what we can do here is we can refer everything back to a standard curve. Okay? You see the mean here of 2100 and it is marked off in divisions of 100. (Figure 3)

But we can refer everything back to a standard curve where the mean is zero and the standard deviation is one. And you do that by using z scores. You take your value, subtract the mean divide by the standard deviation. And that allows us to look things up in a table then. We don’t have to have a table exactly for our purposes though. The 1950, to get its z score I just plug it into x that’s there, minus the mean divided by the standard deviation and that’s — 1.5. Which means that it’s about right there. And what I am looking for actually is the area under this curve in that shaded blue region. Now I look at this table and I get this 43.3% and then half the distribution would be 50%. It’s a nice and symmetric number. So that adds up to be about 93.3%. So there is my prediction – there is a 93.3% chance that that light bulb will last those hours. (Figure 4)

We sometimes assume all tests have to be normally distributed and they don’t. I can give you an example of one that is not. What if I give a test and I put one particular type of problem on there, and you have to know the formula to get the problem? Well, then I
Almost nothing is exactly normally distributed. Amounts are tallied and the frequencies plotted above the amounts, and they assume a curve that is approximately “bell” shaped.

IQ Tests
\[ \bar{x} = 100 \]
\[ s = 15 \]

Figure 2, Segment 1

**Example** - Suppose that a brand of light bulb lasts on average 2100 hours, with a standard deviation of 100 hours. What is the probability that a particular bulb will last more than 1950 hours?

Figure 3, Segment 1
would end up with a distribution something like this, where I’ve got kids who did not know the formula who scored here and then I’ve got kids who did know the formula who scored here and got them all right. That is certainly not normally distributed, okay? (Figure 5)

One thing that we do in our textbooks a little bit wrong is that we claim that biological things are normally distributed. And technically they are not and I will show you why. Here are some heights of U.S. males and it does have that curve. I will show you how that kind of curve kind of fits there. Standard deviation of 70 inches with a standard deviation of 5 inches and it seems to fit, but remember this – that weight is proportional to volume and a change in length causes a change in volume by a factor of $v^3$. It’s a cubed factor because you have that dimension, right, length, length, length gives you the volume. (Figure 6) So if you take the data here, and I’m taking these values right along here, and I’m going to scale things the proportion I am going to use is going to make 70. It kind of goes to 170 I think that is was, which is right about there. Again it stretches it. It stretches it non-proportionally along the axis. So even though it looks like, I went ahead and calculated the mean and it turned out to be about 173 pounds with a standard deviation of 37 pounds. It kind of looks like the curve but you can tell we miss it here, there, and then there. This is actually called the log-normal distribution that works on this and I am not going to get into this because it’s above our means. There is a thing called the log normal and that is the kind of distribution where, if I where I do the kind of cubing of the data and look at the log-norm distribution, I will still have a log norm distribution. So I just want to correct a little misgiving. We’re close. I mean it’s close enough for jazz. It’s close enough to work some problems. (Figure 7)

This is contrary to what you see in many general mathematics textbooks, but think it
We sometimes make the mistake of assuming that all test results are normally distributed, but tests can have any type of distribution.

Suppose that a test has only one type of question, that requires knowledge of a certain formula.

Figure 5, Segment 1

It is often claimed that the heights of US males is approximately normally distributed. Recall that weight is proportional to volume, and that changes in length by a factor of $r$ change the volume by a factor of $r^3$.

Figure 6, Segment 1
through: if a biological measurement like height is normally distributed, then any biological measurement tied to height would be normally distributed as well. We have already demonstrated (Episode 7, Segment 1) that if length grows by a factor of $k$, then volume grows by a factor of $k^3$, and weight is proportional to volume. The probability density function of a normal distribution of mean $\mu$ and standard deviation $\sigma$ is

$$f(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$  

However, if we look at $f(x^3)$, it is immediately clear that

$$cf(x^3) = \frac{c}{\sqrt{2\pi} \sigma} e^{-\frac{(x^3-\mu)^2}{2\sigma^2}},$$  

where $c$ is chosen so that $\int_{\mathbb{R}} f(x^3)dx = 1$, is not a normal distribution. However, with the log-normal distribution, it is the natural logarithm of the (now necessarily positive) data that is normally distributed. Since $\ln k^3 = 3 \ln k$, then the logarithm of the cube of the data will also be normally distributed and thus, the cubed data will have a log-normal distribution.

*I went through some stuff quickly there, so let me flash up some summary pages that kind of summarize what I was talking about.*
**Summary**

**Normal distributions** are useful when data is expected to be symmetric about an arithmetic mean.

![Distribution Diagram](image)

- 68.3%
- 95.5%
- 99.7%

Summarized page 1, Segment 1

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**Summary**

Any normal distribution can be scaled to a standard normal curve ($x = 0$, $s = 1$) using the z-score.

$$z = \frac{x - \bar{x}}{s}$$

Summarized page 2, Segment 1
Summary

Examples of data sets that are (approximately) normally distributed:
- Mechanical error measurements
- Measures of mechanically filled packages
- Lifespans of consumable items (light bulbs, lighters, etc.)
- Standardized test scores (ACT, SAT, IQ tests, etc.)

Summary page 3, Segment 1

Summary

Examples of data sets that are not normally distributed:
- Biological measurements (height or length, weight, etc.)
- Some test scores
- Data that is not randomly sampled (bad survey data)
- Financial indicators (interest rates, stock values, commodity prices, etc.)

Summary page 4, Segment 1
2 Divisibility theorems – Developing a number sense

Objective: To provide the student with some simple techniques for determining divisibility of whole numbers to better prepare them for day-to-day mathematical operations. The concepts of divisibility, prime, composite, and relatively prime are introduced.

The second thing I would like to show you today is hopefully something that you already know. I know that in my education career somewhere in elementary school, I had a teacher show me these things that are really cool little tricks and very helpful little tricks for doing things like simplifying fractions and then later in it was good for factoring things like polynomials and business like this. I want to talk to you about divisibility theorems, and ways of looking at a number without actually having to divide things out and being able to tell that number is divisible by 2, that number is divisible by 8, that number is divisible by 5, and so forth. That is a very useful skill. And I don’t want to get involved in any big and involved examples, applications where you need to know that stuff. But I think you are going to see that it falls into place in a lot of places.

You know if I was looking at this fraction \( \frac{a}{b} \) and I am wondering “Okay, when will this simplify into a whole number?” Okay, well, it’s going to happen that the denominator divides cleanly into the numerator. Or we simply just say that it divides the numerator, so that is the kind of thing I am going to look into how can I look at this a and tell automatically that \( b \) will be able to divide into that. It’s useful to us because sometimes I want to use whole number answers. And that has to happen. And if I can look at that and tell that it doesn’t happen then I have a problem. And these are really easy tricks and when I say grade school tricks, they really are. That is when I was introduced to them, so you’re not going to see any really really tough mathematics here. I will start with the easiest ones to show you a little bit about where they came from, but some of them I’m just going to show you the theorems.

We have several “last-digits” theorems. No matter how many digits a number has, a numeral for the number has, we can express it as just a list of these digits. And what we do is take all but the last one and say okay you got that times a multiple of ten that makes sense. Then you have the last one plus the last one. And really that is how our number system works – its a base-ten numeral system and so I can take those things as a multiple of ten and just add the last one. Okay? Now if I was concerned about this great big multi-digit numeral here being divisible by ten. Well I got a piece of this that is divisible by ten already. Because it is a multiple of ten, alright, that has to be divisible by ten because I can cancel the tens out, for goodness sakes.

So it really comes down to if I look at these two numbers, this one and this one. This will be divisible by ten if and only this is divisible by ten. And the only single digit divisible by ten is zero. That’s it anything else if I divide by ten I get a fraction. Okay so that gives us a quick and easy way to spot divisibility by ten. Just look at the last digit. If it’s zero, it’s divisible by ten. Bet you knew that one already. Okay? (Figure 1)

Okay I am going to get a little tougher. I am just starting with the easy ones. The same idea applies to divisible to by five. Okay, this is already divisible by five so this will be if and only if this is and what single digits are divisible by five? Well, zero and five. And that’s it. Right? What I am asking is what numbers can be divided by five and still get a whole
number. Zero and five, and that’s it. Okay? So a number is divisible by five if and only if its last digit is a zero and a five. Bet you knew that one already too. (Figure 2)

Divisibility by two comes from this same concept. Being a multiple of ten, it’s already divisible by two, so this is equivalent to being divisible, this number being by divisible by 2. So what single digits are divisible by two? Well, the even ones, right? Zero, 2, 4, 6, and 8. Okay? (Figure 3) So for each of those three numbers all we have to do, I don’t care if there is a hundred and twenty of these numbers in front of it, look at the last one. And that last digit will tell us then if it’s if it’s divisible by ten, two and five. Okay?

I might try to pull this stunt will 4 but let me show you the problem here. As a multiple of ten this is not necessarily divisible by four. It could be but it doesn’t have to be. However, there is a power of ten divisible by four. There’s a 100. If I take all of the last two, I could say that times a hundred and take the last two and put those two together. And then I have the same basic set-up. This as being a multiple of hundred is divisible by four automatically, so this will be divisible by four if the last two digits are divisible by four. So you can have a ton of them, I don’t care, I will just look at the last two. Probably I know I should know my multiplication rules by now. I can tell if that is divisible by four much quickly than dividing the whole number out, right? (Figure 4)

Eight is similar but 100 is not necessarily divisible by 8, so I bump it up to a 1000, and I look at the last three digits right there. So a number is divisible by 8 if and only if its last three digits are divisible by 8. And I’m not going to take the time to change it but if I wanted to go to 16 then I would have to look at the last four digits. And see if those last four were divisible by 16 and then it’s starting to get a little messy. But that is the idea that I can look

"Last Digit" Theorems
No matter how many digits (over one) a number has, we may express it as
\[ d_1 d_2 \ldots d_{n-1} d_n = d_1 d_2 \ldots d_{n-1} \cdot 10 + d_n \]

- A number is divisible by 10 if and only if its last digit is a 0.

\[ d_n = 0 \]
“Last Digit” Theorems

No matter how many digits (over one) a number has, we may express it as
\[
d_1 d_2 \ldots d_{n-1} d_n = d_1 d_2 \ldots d_{n-1} \cdot 10 + d_n
\]
divisible by 5
\[
d_n = 0,5
\]

- A number is divisible by 5 if and only if its last digit is a 0 or a 5.

Figure 2, Segment 2

“Last Digit” Theorems

No matter how many digits (over one) a number has, we may express it as
\[
d_1 d_2 \ldots d_{n-1} d_n = d_1 d_2 \ldots d_{n-1} \cdot 10 + d_n
\]
divisible by 2
\[
d_n = 0,2,4,6,8
\]

- A number is divisible by 2 if and only if its last digit is a 0, 2, 4, 6, or 8.

Figure 3, Segment 2
at those last digits and I can avoid all the previous digits. Okay? (Figure 5)

We also have some sum theories and these are really cool, where I can actually look at the sum of all the digits and tell if a number is divisible by nine, for example, or three. Um A number is divisible by three if and only if the sum of its digits are divisible by three. So this bug-ugly thing rather than have to divide it out to see if I get a whole number, I can really just add up the digits. 14, 16, 25, 32. Thirty-two is not divisible by three, right? In fact, when I check this I get a remainder of two. And then when I check this, I get a remainder of two as well. (Figure 6) The cool thing is if I bump up any one of those numbers and it doesn’t have to be the last one, any one of those, let me change this watch this, let me change the four to a five. And change this four to a five. And that makes that a 33, and all of a sudden my number is divisible by three. It’s really slick how that works and you can prove that but it’s a little bit messy – I didn’t want to get into it. (Figure 7)

Nine works in exactly the same way. I can look at this number and tell you that this 7,352,000 blah blah blah is not going to be divisible by 9 because of the 33 is not divisible by nine. (Figure 7)

The “little bit messy” proof goes as follows: Suppose that the n-digit numeral

\[ N = d_1 \times 10^{n-1} + d_2 \times 10^{n-2} + \ldots + d_{n-1} \times 10 + d_n. \]

Then

\[ N = d_1(10^{n-1} - 1) + d_2(10^{n-2} - 1) + d_2 + \ldots + d_{n-1}(9) + d_{n-1} + d_n. \]
**“Last Digits” Theorems**

\[ d_1 d_2 \ldots d_{n-2} d_{n-1} d_n = d_1 d_2 \ldots d_{n-3} \cdot 1000 + d_{n-2} d_{n-1} d_n \]

- A number is divisible by 8 if and only if its last two digits are divisible by 8.
- A number is divisible by 4 if and only if its last two digits are divisible by 4.
- A number is divisible by 16 if and only if its last four digits are divisible by 16.

**Figure 5, Segment 2**

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**“Sum” Theorems**

- A number is divisible by 3 if and only if the sum of its digits is a divisible by 3.

**Example**  Is 7,342,952 divisible by 3?

- \[ 7 + 3 + 4 + 2 + 9 + 5 + 2 = 32 \] not divisible by 3
- \[ 7,342,952 + 3 = 2,447,650 \] R 2

**Figure 6, Segment 2**
“Sum” Theorems

- A number is divisible by 3 if and only if the sum of its digits is a divisible by 3.

**Example** Is 7,352,952 divisible by 3?

\[ 7 + 3 + 5 + 2 + 9 + 5 + 2 = 33 \quad \text{divisible by 3} \]

\[ 7,352,952 + 3 = 2,450,984 \]

7,352,952 is not divisible by 9 since 33 is not

- A number is divisible by 9 if and only if the sum of its digits is a divisible by 9.

Figure 7, Segment 2

and

\[ N = d_1 \left( 9 \sum_{k=0}^{n-2} 10^k \right) + d_2 \left( 9 \sum_{k=0}^{n-3} 10^k \right) + \ldots + d_{n-1}(9) + (d_1 + d_2 + \ldots + d_{n-1} + d_n), \]

so that

\[ N = 9 \left( d_1 \sum_{k=0}^{n-2} 10^k + d_2 \sum_{k=0}^{n-3} 10^k + \ldots + d_{n-1} \right) + \sum_{k=1}^{n} d_k. \]

Therefore, since the first term is always divisible by 3 (and 9), then N will be divisible by 3 (likewise, 9) if and only if \( \sum_{k=1}^{n} d_k \) is divisible by 3 (likewise, 9).

Okay and then the last one. What really opens things up for the rest of them is I can look at products of numbers as long as they are relatively prime. What I mean by that is they have no common prime factors. For example 4 and 9, this is made up entirely of 2’s and this is made up entirely of 3’s, so they are relatively prime. Four and 6 are not, because they share a two factor. And as long as they are relatively prime then a number will be divisible by the product if it is divisible by both. (Figure 8) So for example I can tell you that this number is divisible by six, because it is divisible by two. 4 right? It’s divisible by 3, because I add them all up and I get 27, which is divisible by three. And 2 and 3 are relatively prime, So there you go, you’ve got a theorem there for six. Also, if you look at it, divisible by four and nine because if you look at the last two it’s divisible by four. 27 is divided by nine so it would also be divisible by 36. Trust me you would not want to divide that by 36 to figure that out. (Figure 9)
“Product” Theorem
Two numbers are relatively prime if they have no common prime factors.

Example
\[
\begin{align*}
4 &= 2^2 & 9 &= 3^2 & \text{relatively prime} \\
4 &= 2^2 & 6 &= 2 \cdot 3 & \text{not relatively prime}
\end{align*}
\]

- A number is divisible by \(ab\) if and only if \(a\) and \(b\) are relatively prime and the number is divisible by both \(a\) and \(b\).

Example 63,104,364 is divisible by 6
63,104,364 is divisible by 2 (last digit)
63,104,364 is divisible by 3, since
\[
6 + 3 + 1 + 0 + 4 + 3 + 6 + 4 = 27 \text{ is divisible by 3}
\]
Then, since 2 and 3 are relatively prime, 63,104,364 is divisible by 6.

It is also divisible by 36, since it is divisible by 4 and 9, and they are relatively prime.
Okay, so these are some good little tricks that you can use and I have summarized them for you on a few pages that I will flash up now.

**Summary**

“Last Digit” Divisibility Theorems:

- A number is divisible by 10 if and only if its last digit is a 0.
- A number is divisible by 5 if and only if its last digit is a 0 or a 5.
- A number is divisible by 2 if and only if its last digit is a 0, 2, 4, 6, or 8.
Summary

“Last Digits” Divisibility Theorems:

- A number is divisible by 4 if and only if its last two digits are divisible by 4.
- A number is divisible by 8 if and only if its last three digits are divisible by 8.
- A number is divisible by 16 if and only if its last four digits are divisible by 16.

Summary page 3, Segment 2
Summary

Two numbers are relatively prime if they have no common prime factors.

**Example**

\[
\begin{align*}
4 &= 2^2 & 9 &= 3^2 & \text{relatively prime} \\
4 &= 2^2 & 6 &= 2 \cdot 3 & \text{not relatively prime}
\end{align*}
\]

**“Product” Divisibility Theorem:**

- A number is divisible by \(ab\) if and only if \(a\) and \(b\) are relatively prime and the number is divisible by both \(a\) and \(b\).
3 Number of polynomial solutions and end behavior – Modeling volume

Objective: To show how understanding the behavior of polynomial graphs can help us to better use our graphing calculators to solve problems.

Last thing I would like to talk to you about today is the behavior of higher degree polynomials. By higher, I mean higher than two. Degree-two polynomials are quadratics, we know pretty much how they behave and we can do things like solve quadratic equations exactly using the quadratic formula. We can find the max or the min – it will only have one of those two – by finding the vertex. We really are pretty good with quadratics. But when we get to polynomials that have a higher degree then we run into some trouble. We don’t have a cubic formula or have access to one that we are going to use all the time. In fact, you get so high, there is no formula to solve for at all. So we get a greater dependence then on our graphing calculators until such time that we can get to calculus and learn how to do some of these tricks symbolically when polynomials of a degree 5 and higher pop up.

When we do volume calculations, we are probably going to end up with degree-three polynomials. Weight is proportional to volume, so it will pop up there. Sometimes we will solve radical equations where we’ve squared both or raised both sides of the equation to a power and higher powers will pop up then. And even some things will circular measure you can put these things together and you end up with some cubics and quadratics. So it is important that we understand how these things behave because we are going to have to put them on our calculator.

And we have to answer this question then. Are we looking at all the interesting stuff? You know, you just have that little viewing rectangle that you use on your graphing calculator. How do you know that is at the good stuff? There may be something really interesting happening over here that you are just not looking at. So I just want to talk to you for a moment about behavior and how to tell if you are looking at the whole quote-unquote graph. All the interesting stuff with a polynomial.

The first little trick we can keep our eyes open for is called the fundamental theorem of algebra. That just says that a polynomial of degree n will have at most n real zeros. If you want to get technical about it, it will have n complex zeros but we are worried most about those that will appear on our calculator. So here is a degree-five polynomial and sure enough it hits the x-axis, these are the zeros that I have circled, 5 times. Right? (Figure 1) In fact, you will notice that it has four peaks and valleys. You know that is the most peaks and valleys it could have, but beyond that this, I mean, it doesn’t have to have five zeros. For example, if I scaled it upwards, okay, this only has one real zero, but you can see if I drew a horizontal line across there that I could hit it five times and that is what you really need to look for how many times could a horizontal line pass this thing. Okay, so that tells me all the possible zeros are right there. I’m looking at them okay? (Figure 2)

Another thing to take into account is end behavior of the graph. If a polynomial is even degree, then it doesn’t matter if we take a big positive number or a big negative number and raise it to an even power, you are going to get a big positive number. So as long as the leading coefficient in front of that term is positive, it will stay positive and both edges will go
**Are we looking at the entire graph?**

**Fundamental Theorem of Algebra**

A polynomial of degree $n$ will have at most $n$ real zeros. (It will, in fact, have $n$ complex zeros.)

![Degree-5 Polynomial Graph](image)

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**Figure 1, Segment 3**

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**Are we looking at the entire graph?**

**Fundamental Theorem of Algebra**

A polynomial of degree $n$ will have at most $n$ real zeros. (It will, in fact, have $n$ complex zeros.)

![Degree-5 Polynomial Graph](image)

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**Figure 2, Segment 3**

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up like this. It’s good! Okay? But if the $a$ is negative, it will take those positive values and turn them down and make them negative. So I just flipped it over really and that leading coefficient is a negative it will flip it down. With even degree they are both going to go in the same direction. (Figure 3)

**Are we looking at the entire graph?**

**End Behavior**
The ends of a polynomial of even degree will point in the same direction, up if the leading coefficient is positive and down if it is negative.

$$y = ax^n + \ldots$$

- $a > 0$
- $a < 0$

(Figure 3, Segment 3)

Now with odd degree different things happen. You take a positive number, raise it to an odd power, and it stays positive. Let me get this right, here. But if you take a big negative number and raise it to an odd power, then it is negative so you get this kind of John Travolia kind of thing. As long as the $a$ is still positive, they don’t change the signs, like this, okay? Which is what I think I am doing here. Then if the $a$ happens to be negative, it will flip it around and you’ve got things, you got John Travolia over to this side, okay? Odd degree, different ends. (Figure 4) Okay, that works – you kind of spot, if you are graphing something, there’s odd degree but in your graphing window but ends are going in the same direction, you need to expand your window. You don’t have everything you need to see there.

Okay, so let me give you an example. This is just something you could be doing in your workshop or something. Say you want to construct an open box to keep tools in out of a 16 by 20 inch piece of sheet metal, and you are going to do that by cutting square corners out, and then folding the sides up and welding the cuts so it looks something like this. Of course, it would be height $h$, I’m sorry, I mean $x$ when you do that. Okay, what I would like to do is worry about the volume of this. If I’m going to build this box, I would like it to have as much volume as possible so I can put stuff in there. Well, what you are doing here is you have the 16 this way and I didn’t put the 16 this way. That’s the short side. And then you are removing $x$ from both sides so this base would be $16 - 2x$. Okay whatever $x$ is, this will be
16 – 2x, and then you have 20 up and down and you are taking x away from there and there, so this dimension would be 20 – 2x. And so the volume, length times width times height, is just this. And that is a degree three polynomial. That’s higher than quadratic and it could be trouble if we try to solve equations with that. (Figure 5)

Just to show you what it looks like let me first multiply it out and then graph it so this is a odd polynomial. Notice that the endpoints are doing the old John Travolia thing, okay? And then because that leading coefficient – I’m cracking up the camera operators, this is great – the leading coefficient is positive, so it goes up on this side and goes down on this side, okay? And if any horizontal line were going through there it would hit three times. You know, I could put a horizontal line through there three times. So this is all the activity I want to see in the graph. (Figure 6)

And then I can, from that graph, I can do lots of things. I can say, “Well, which x’s are valid?” Well, I can’t have negative volume here, folks, right? So, where this dips down below the x-axis, that’s bad. I can only go out here to 8 and then which x gives the maximum volume? Well, until I know some calculus, I’m just going to have to rely on my calculator to find that peak, there in the valid range. (Figure 7)

Using calculus, we can find the exact answer by setting \( V'(x) = 12x^2 - 144x + 320 = 0 \) and solving for \( x \) in the range \([0, 8]\). The exact maximum in that range is \( x = \frac{2(9 - \sqrt{21})}{3} \).

And then I could ask the question, you know, let me set the volume at 400 cubic inches I can expect two answers and really you can try to find them by factoring. In fact, all these numbers are divisible by 4, you know, I’ve got my nice divisible theorems to fall back on. But
Example: Suppose that you are in your metal shop constructing an open box from a 16 in by 20 in piece of sheet metal by cutting four equal size squares out of each corner and folding up the sides.

\[ V = x(16 - 2x)(20 - 2x) \]

Figure 5, Segment 3

Example: Suppose that you are in your metal shop constructing an open box from a 16 in by 20 in piece of sheet metal by cutting four equal size squares out of each corner and folding up the sides.

\[ V = x(16 - 2x)(20 - 2x) = 4x^3 - 72x^2 + 320x \]

Figure 6, Segment 3
I don't know how to factor this. I have to rely on my calculator to get those answers. So when you get above degree two, that calculator is going to be your best friend. It’s going to find those answers that otherwise you would not be able to find. (Figure 7)

Let me summarize what I was just talking about, and I will be back in just a second.

Closing

Well, any day that I get to impersonate John Travolia is a good day. I really enjoyed the stuff we talked about today. I hope you find it useful, and I hope you find it helpful in your work. Let me remind you that the things you saw there, if you don't catch in on TV or you want to go back and check your facts, all these files appear on our webpage and you can download them. It’s a big download, but it might be worth it. They are listed by the content areas. Also on that page, you will find a place where you can email me. Please do that, let me know how things are going, and let me know if you have some math that you have seen that you would like me to try to find an application for. But please contact me, let me know. With that I’m done. Thank you so much for watching and we will see you next time.
Summary

Fundamental Theorem of Algebra
A polynomial of degree $n$ will have at most $n$ real zeros. (It will, in fact, have $n$ complex zeros.)

Summary page 1, Segment 3

Summary

End Behavior
The ends of a polynomial of even degree will point in the same direction, up if the leading coefficient is positive and down if it is negative.

$y = ax^n + \ldots$

Summary page 2, Segment 3
Summary

End Behavior
The ends of a polynomial of odd degree will point in opposite directions, with the positive end up if the leading coefficient is positive and down if it is negative.

\[ y = ax^n + \ldots \]

\[ a > 0 \quad a < 0 \]

Summary page 3, Segment 3

Summary

For a polynomial of degree < 2, we may be able to find exact zeros if the polynomial will factor, or, if we know one or more exact zeros, by synthetic division.

For a polynomial of degree < 2, we can not find exact maximums or minimums without calculus. Approximate, numerical max and min’s can be found using graphing calculators.

Summary page 4, Segment 3