1 Graphs of data – Misleading tables and graphs

Objective: To illustrate the importance of knowing how to interpret graphical representations of data, by showing how they can be used to mislead the viewer.

Hello, welcome to Math Matters. I’ve got a little something for everybody today, so why don’t we just get right after it? I want to start talking today about graphical data representation. You’re taking data and putting some kind of picture, we see this a lot in newspapers, uh, when, when a survey has been taken or, or some information has been gathered, they’ll put this in the newspaper in some kind of nice graph and you have to be careful about this because they can mislead you by the way they present the data. So I want to look at some examples, today, of misleading data. There’s actually a lot of ways to lie using statistics. There’s at least one book I know of devoted to it, “How to Lie with Statistics” by Darrell Huff. It was required reading back when I took statistics and it has a lot of things mentioned in there and some of it was computational and some of them are ways to manipulate the data in such a way that it misleads. But the ones I want to focus on today are simply the graphical kind of things that you can do to try to get in your, tricking your eye into tricking your brain I guess is the idea so lets look at an example.

One trick that they can, people can use, or folks can use, to mislead you is they can use a narrow range in graphs. Now suppose you have a, uh, a data set where the change over a period of time is very small. I’ll just flash up some data that’s basically flat you know from 2002 to 2006 there’s not much change there. (Figure 1) Well, watch what happens if I change, I am going from zero to 10 along the y-axis here, watch what happens when I change the range on that axis now. If I change it from 9.8 to 10.3, then all of a sudden it looks like it’s increasing dramatically. You, there’s something, something bad is happening or something good is happening and you have to convince the public about this. It’s the same data – the only thing that has changed is the the way we position things along this y-axis so you’ve gotta be careful about this, you’ve gotta keep your eyes open for problems like this where they kind of go through and there there not. One tip off is most of these things you start at zero down here at the bottom. If they don’t then there is a good chance you are being manipulated. That’s, that’s one of the oldest tricks in the book. (Figure 2)

I can take it one step farther and completely remove marks from the graph, whatsoever. Have you ever seen an ad something like this? This is my dramatic TV ad “Our customer satisfaction ratings are going through the roof!” and they even, and I like this little part, and they even put a little arrow here to say “Yeah, yeah it even went higher but we got tired of drawing you, this kind of thing.” But if you’ll notice there are no markings on this graph whatsoever, so what is this telling you? What is it telling you? What are we measuring through here? Time? I don’t know. Cars sold? I don’t know. And then on the other axis there is no other markings. Satisfaction? How do you measure satisfaction? What do you
**Narrow Range in Graphs**

Suppose that the change in a set of data over a period of time is very small.

![Graph](image1)

**Figure 1, Segment 1**

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**Narrow Range in Graphs**

Suppose that the change in a set of data over a period of time is very small.

Does this look more dramatic?

*Same data!*

![Graph](image2)

Keep your eyes open!

![Graph](image3)

**Figure 2, Segment 1**
give? A survey? Are they counting the number of people who didn’t report them to the Better Business Bureau? And what’s the scale? Did they start at zero? I don’t know. There there’s no way to gauge what’s going on this graph whatsoever. And that is uh something to kind of watch for. That’s a ti-that’s a tip off right there if there is no markings then you have the graph is meaningless. And I even did, I even went one step farther and made the graph narrow so that it even looks like it’s even steeper. You know that the increase is even steeper. Just a couple things to watch for. (Figure 3)

Another little trick that they can use, too, or that folks can use to mislead you, is the idea of showing linear change but showing it with volume. Now I know some things like USA Today are very creative with their graphs, they do a lot of these kind of things, and it’s okay as long as you just change one dimension. Let me give you an example.

I don’t know exactly what I am measuring here but in 2005 there was one of these things so I am using a three dimensional bar to indicate that. Now in 2006 it doubles. Lets say that it doubles. It’ll go up to 2. Now if you’ll notice I kept the dimensions of the base the same the only dimension I changed was the z-dimension, the up and down dimension. And when I doubled that, it actually doubles the volume and that’s, and that’s a true representation of a doubling of the data okay? (Figure 4) Now here’s the way I can mislead you on this that I could double every dimension okay not just the up and down dimension but the left and right and the back and forth dimension that would look likethis. What happens here yes the amount has doubled but if you look at the bottom of this what your eye perceives is the volume of these objects and what is happening when you double each of these dimensions, you’ve got, you’ve doubled it here, you’ve doubled it there, you doubled it there, two times,
two times, two is eight so the volume of the shape shown, the solid shown, is actually eight times the volume of the original shape. And so yes it is two times higher but what your brain picks up is “Wow, that’s a big change!” That’s eight times, a factor of eight change when really, that’s a misleading part of data. (Figure 5)

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**Linear Change Shown With Volume**

Suppose that an amount doubles over one year, and we use a 3-D solid to represent the original amount.

\[ V_1 = xyz \]

\[ V_2 = xy(2z) = 2xyz = 2V_1 \]

Correct!

Figure 4, Segment 1

I’ve got one more to show you this ones a little more computational. Odd shaped pie graphs again USA Today’s great about doing this – a circle graph or a pie graph or pie chart (people use different terminology for this,) but you can get the area of the entire thing simply by taking percents of 360 degrees. So this 20% represents 20% of the entire area and to get that I had to take 20% of 360 degrees and got 72 degrees. I’m getting my angle measures here and again with 25%, it’s easy to calculate these things and it’s a true representation of what percent of the data is going into each category. (Figure 6) A pie graph is a good, is a good picture to use when you are breaking data down into categories. But now what USA Today does or what some newspapers will do is they’ll say, “A circle is so plain. Let’s do something creative. We’re talking about the US dollar, so let’s do a dollar graph.” Or if it’s chicken sold at KFC, then it’s a chicken graph or whatever it is. (Figure 7)

Now, if you’ll notice I didn’t change I didn’t go back I didn’t change any of the angles okay? But because this boundary is no longer equal distance from the center I have changed the percentages that go in each category. Even though I still have it marked at 20 it’s actually not 20%. I had to use a little calculus to calculate this percentage. It’s more like 17 percent, and that’s because it gets out to the boundary a little a little quicker than some of these others do. Instead of 25%, that’s more like 31%. Instead of 40%, this is more like 43% of the entire area. And the last one, that’s more like 9%. That’s the kind of mistake that could be made
**Linear Change Shown With Volume**

Suppose that an amount doubles over one year, and we use a 3-D solid to represent the original amount.

\[ V_1 = xyz \]

\[ V_2 = (2x)(2y)(2z) = 8xyz = 8V_1 \]

Looks 8 times bigger!

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**Odd-Shaped “Pie” Graphs**

Area percents correspond to percent of a full revolution (360°)

- 0.15(360) = 54°
- 0.2(360) = 72°
- 0.4(360) = 144°
- 0.25(360) = 90°

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Figure 5, Segment 1

Figure 6, Segment 1
inadvertently by a journalist or a graphic designer that didn’t pay enough attention in their math class – didn’t learn these little tricks. But it could also be used to mislead you. Those two categories where the actual percentages are less you could come back and say “Hey, I told you it was 20%, but what you told my eyeballs what that it was less than that, okay?” So it can be used to mislead, and that’s the thing I’m kind of watching for. (Figure 8)

I’m going to flash up some summary slides that kind of talk about these things, and then we’ll move on to the next topic.
Odd-Shaped “Pie” Graphs

Area percents correspond to percent of a full revolution (360°)

![Pie Chart Image]

Figure 8, Segment 1

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**Summary**

Be wary of . . .
- graphs with an extremely narrow range along the vertical axis that may visually “increase” the rate an amount is growing.
- graphs of amounts that are hard to quantify, and that have no markings on the axes at all.

Summary page 1, Segment 1
Summary

Be wary of . . .

• graphs that express “1-D” changes in data with a “2-D” or “3-D” shape. A linear change by a factor of \( r \) will cause a change in area by a factor of \( r^2 \), and a change in volume by a factor of \( r^3 \).

• adapted “pie” graphs where the angle measure percentage of 360° does not match the percent of the total area.
2 Volume – Home-improvement projects with concrete

Objective: To illustrate how volume formulas are relevant to our everyday lives by examining a real-life example of their use, specifically in calculating how much concrete is needed in different home-improvement projects.

Another topic where I frequently hear “Why do I need to know this?” is sometimes when I've taught general math or math for elementary school teachers we talk about three dimensional solids and their surface area and their volume and a lot of times they say “Why do I need to know this, this kind of stuff?” The truth of the matter is a lot of applications for just home improvement are jobs where you have to be able to calculate volumes to do things. You could, if you’re blowing insulation into a wall, you have to have some knowledge of how much insulation you are gonna need to put in there. If you are buying mulch, you are gonna need some kind of calculation of how many cubic yards of mulch to buy – these kind of things. Maybe you are digging out a decorative pond and you need to know how much dirt you’re gonna need to fill in around it. There’s all kinds of things where you, where you need to figure volume.

Now I do want to go ahead and caution you a little bit on this. There’s a difference. We talk about volume as a a cubic measure of length. We also talk about a lot of times capacity when we mention things like cups, pints, gallons, liters, these kind of things. There’s a difference in the two. They are related, okay, and the best way I think to illustrate that is by thinking of a brick, okay? A brick has volume – you can measure it in three dimensions. It’s a, it’s a block, so you can calculate the volume. It doesn’t have capacity. It might have a little capacity if you fill in the holes in the brick. But if it’s a solid brick, it won’t have any capacity. Capacity is how much liquid basically something can hold. Volume is how much space does it take up, okay? So I’m working on volume, so I’m not worried about liquid kind of containers and these kind of things.

The one application I want to show you today that I think will be interesting is pouring concrete. I’ve done this as a kind of do-it-yourself-er at home. I’ve taken on projects like this, where I have poured my own concrete. You might even wind up in a job where you do this. I mean, you could do this do that for a living, and those folks, a lot of times, don’t have college degrees. So I get a little touchy when you start to say “Why do I have to know this?” Well, there’s people out there without college degrees who have to know it to get their job done. So what I would like to do is look at a few different situations where you would have to, if you were gonna pour this, or you would have to give an estimate on getting it poured, or something like that, you would have to calculate volume.

Let’s say you are gonna pour a sidewalk kind of in an “L” shape – 12 feet in this direction, 15 feet in this direction. It’s a 4-foot sidewalk and these little gray lines, they put in little breakers in there to allow for expansion. We could calculate the area of this. We don’t know how to do the area of an “L”-shaped thing, but we can break it up into pieces, right? If you notice, I have put in a little purple line here. We could break it up into a 12 by 4 rectangle and then an 11 – that’s 15 – 4 – rectangle and it has an area of 92 feet on the top, okay? (Figure 1) But what we are really looking at is pouring this thing. Typically, they pour concrete four inches deep. This is a prism, okay? We have a polygon, the base of this thing, and the volume of prisms is pretty easy. It’s the area of the base times the height. The only
thing I have to be careful about is I have one measure in feet I have another measure here in inches, and I need to use the same measure throughout. So I would go 92 but then four inches is \(\frac{1}{3}\) of a foot so that is \(\frac{92}{3}\) cubic feet. But then concrete is sold by the cubic yard, so I need to do a conversion there. But again, it's multiplying by one three times. One yard is three feet, one yard is three feet, one yard is three feet. So it works out to be a little over one cubic yard for that kind of pour. That's a basic kind of arithmetic, but that's a useful kind of thing for those kind of projects. (Figure 2)

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Here's one that's a little tougher, perhaps. Say that you are pouring a ramp up a outbuilding or something like that, or a storage building, and it's going to be 4 by 6, and it's going to have an increase of steepness by 8 inches. You are going to pour this in concrete. The thing to kind of remember is, you can't just pour this on top of the ground. You have to have four inches of base underneath here, so to calculate this volume again, this is a prism, this is a polygon on the end here, and it goes consistent throughout to the other side. It's already broken into two pieces here, so I can calculate the base – area of the base is one half base times height – that is the triangle. So, that's one half 4 and then eight inches – this is \(\frac{2}{3}\) of a foot. And then it's a four foot by four inch rectangle so that's four times \(\frac{1}{3}\), and that works out to be \(\frac{8}{3}\) square feet. Then the height of the prism in this case is six feet so multiply by 6 – that's 16 cubic feet. And again we can calculate the volume in cubic yards. One over 27 cubic yards is, that's the result of saying 3 times 3 times 3 in the denominator. So it works out to be almost \(\frac{6}{10}\) of a cubic yard, okay? (Figure 3)

Now I meant to mention this earlier. When you are ordering concrete, you probably ought to round up, because once you start working with concrete, you really can't stop and go back.
Sidewalk

4 in

Volume = $92 \left( \frac{1}{3} \right) = \frac{92}{3} \text{ ft}^3$

The form is a prism, with area of the base $92 \text{ ft}^2$.

$V = Bh$

$\approx 1.14 \text{ yd}^3$

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Ramp

8 in

$V = Bh = \left( \frac{8}{3} \right) \cdot 6 = 16 \text{ ft}^3$

Again, the solid is a prism, with

$B = \frac{1}{2} (4 \left( \frac{2}{3} \right) + (4 \left( \frac{1}{3} \right) = \frac{1}{3} \cdot \frac{16}{27} \text{ ft}^3$

$\approx 0.59 \text{ yd}^3$

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Figure 2, Segment 2

Figure 3, Segment 2
and do it later, so it’s probably wise to round up. Typically they carry a little extra onboard just in case, but it’s probably wise to round up just in case.

Let me give you one more shape. I’ve see some patio’s like this – kind of decorative things where you have a circular patio maybe have a grill set up around the outside of it, and some plants and some things like that. The one I am looking at has a radius of eight feet, or if you want to think of it, a diameter of 16 feet across. Again, I would like to pour this at least four inches deep. When you look at it like this you have to realize, this is a cylinder, right? It has a circular base, it has a circular top, it’s consistent throughout. And again it’s base times height, but in this case the base of a cylinder is always a circle, so it’s \( \pi r^2 \) – that’s the area of a circle – times the height. So with our calculation here, the radius is 8, square that, \( \pi \) and then again the height in terms of feet would be \( \frac{1}{3} \). So this works out to be \( \frac{64\pi}{3} \) cubic feet. The last thing we need to do is that conversion again to cubic yards, because that’s the way you order things, and it works out to be about 2.48 cubic yards. Which is actually a pretty big pour – circles take up a lot of space. (Figure 4)

![Circular Patio diagram](image)

**Figure 4, Segment 2**

That’s just one example and we could have gotten into mulch and things like that, but concrete made the most sense to me, because you can pour concrete with minimal waste. You can also get into situations where you can pour it into a very complicated shapes and you have to be able to adapt your formulas that you know, for boxes and cylinders and things like that, to the situation. I gave you some examples of doing that. Again, I am going to flash up some summary pages of the things that we talked about and get ready to talk about the stuff in the last section.
Summary

1. **Rectangle**
   - Area: $A_{\text{rectangle}} = lw$

2. **Triangle**
   - Area: $A_{\text{triangle}} = \frac{1}{2}bh$

3. **Circle**
   - Circumference: $C_{\text{circle}} = 2\pi r$
   - Area: $A_{\text{circle}} = \pi r^2$

Summary page 1, Segment 2

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Summary

- **Prism**
  - Volume: $V_{\text{prism}} = Bh$
- **Cylinder**
  - Volume: $V_{\text{cylinder}} = Bh$
  - Where $B$ is the area of the base

Summary page 2, Segment 2
**Summary**

Volume calculations are needed in situations where we have to “fill” a certain 3-D region, as in the concrete-pouring examples.

In order to actually apply the volume formulas we know, we have to be able to adapt them for use when the solids do not necessarily fit the formula.
3 Quadratic formula – Solving non-factorable quadratic equations

Objective: To illustrate the need for knowledge of the quadratic formula by showing how delicate the process of solving by factoring is. We examine one real-world example to show how the quadratic formula can be used.

I hope you’re seeing some good stuff today. These are some cut and dry kind of examples of how the math we teach in those entry-level courses is useful to you on an everyday basis. What I’m going to talk to you about right now is kind of setting up for what we’re going to do throughout the rest of this. I want to talk about the quadratic formula. And I’m not going to focus so much on examples right now, but I do want to justify why we teach this crazy thing and why it’s important for us to know it. In fact, I’m not even going to prove the thing. I’m counting on your teachers to do those kind of nuts and bolts kinds of things. But what I want to get across to you is why do we need to know this ridiculous formula.

Now if you’ve seen this, it’s kind of messy, and those of us who have been in math a long time, we have it burned into our brain. You know, we don’t have to remember it, it’s there. But there’s a purpose behind knowing that, and I kind of want to get into that today. We have talked about things in the past that had this form and let me kind of flash this up. Some constant and then \( x - m \) times \( x - n \), okay, now this would be a quadratic equation, because when I FOIL this out, I’m going to get an \( x^2 \) term. Well, we talked about in the past about how this has solutions at \( x = m \), that makes sense. If I plug in \( m \) here, I get zero, plus the rest of it. Zero times anything is zero, same deal with the \( n \). If I plug in the \( n \), multiply this out, it’s zero times everything else, so I get zero, okay? (Figure 1)

We also discussed how not every quadratic equation, this is the more kind of general form of the quadratic equation. We can’t always write things like this like that. And that’s a difficulty. We start out solving quadratic equations by factoring, but there’s a lot of things that can’t, simply cannot be factored, okay? For two reasons: one of the reasons is the parabola that would be the graph of this quadratic function may not cross the x-axis. Let me just kind of set this in motion. What I’ve kind of done here is I’ve taken a general parabola and I’ve juked around some of the parameters, if you will, to kind of show it. And here you’re looking at somewhere where they do cross the x-axis and there you’re looking at some place where they don’t, okay? Let’s see if I can flash that back. Doesn’t, there it crosses twice, crosses twice, and then eventually it kind of move up. At some point, it crosses just once and then it doesn’t cross at all. And even worse than that, many of those that do cross the x-axis, the second problem is that many of those that do cross the x-axis can’t be factored. (Figure 2)

I’ll give you an example. Here’s one where I can actually solve for the zeroes. If I look at this equation, I need two factors of six that differ by one. Well two and three, I just need to make sure that the three is negative so that I get a negative middle term here. Okay, this is what I call the reverse FOIL method here, okay? So I have solutions then at \( x = -2 \) right there, and \( x = 3 \) right there, okay? (Figure 3) But watch how easy I can mess this up. What if make that six a five, okay? That wasn’t tough. I can still see my zeroes on the graph. They’ve now moved a little bit closer to zero. This is now inside of three, this is the inside of negative two. But what are they? If I go to factor this, I now need two different factors of.
We have previously discussed how quadratic equations of the form
\[ a(x - m)(x - n) = 0 \]
have solutions at \( x = m \) and \( x = n \).

We also discussed how not every quadratic equation of the form
\[ ax^2 + bx + c = 0 \]
can be written in the factored form above.

Figure 1, Segment 3

Many parabolas do not cross the \( x \)-axis.

Many of those that do cross the \( x \)-axis cannot be easily factored.

Figure 2, Segment 3
five that differ by one. I don’t know what those are. Okay, I’m kind of in a quandary to find those. So just that easy, I’m able to take apart that whole idea that we can factor everything and get the answer. It just isn’t the case. We need another method to solve these kinds of equations which pop up in a lot of stuff. (Figure 4)

So we end up using what is called the quadratic formula which I’m now going to flash up for you. Let me warn you ahead of time, if you haven’t seen this thing, it could cause bleeding from the ears because it’s very messy, but now that you’re braced, I think you can handle it. This crazy \(-b\) plus or minus \(\sqrt{b^2 - 4ac}\), all of that over \(2a\), okay? That is a formula that will give us the two solutions to this general quadratic: the “a,” the “b,” the “c” come from the entry of the general quadratic. The “a” is the coefficient of the \(x^2\), the “b” is the coefficient of the \(x\) and “c” is the constant term, okay? And your teacher will derive that for you. I don’t want to, I don’t want to do that here due to time constraints. But it does tell us a bit about what’s going on. (Figure 5)

Because of this square root, we could run into some problems, right? If this \(b^2 - 4ac\) business is positive, then the plus or minus here will give us two real answers. If it’s zero, we’re adding and subtracting zero, so we only get one real answer. And here’s the trouble maker – if it’s negative, then we’ve got that negative under the radical. If it’s a complex number we talked about last time, I’m not going to get any real answers. I’ll get two complex answers. That would be the situation where things do not cross the x-axis at all. They’re floating, the parabola’s either floating above it or floating below it opening downward. (Figure 5)

The one thing I will point out is that if you average the two answers that gives you the axis of symmetry. If I add these things together, the plus and minus negate each other. I get
The solutions to quadratic equations of the form $ax^2+bx+c = 0$ can be found using the **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- If $b^2-4ac > 0$, then we have 2 real $x$.
- If $b^2-4ac = 0$, then we have 1 real $x$.
- If $b^2-4ac < 0$, then we have 0 real $x$.

The average of the solutions is the axis of symmetry $x = \frac{-b}{2a}$.
two of the \(-b\)'s but then when I average, I divide by two and so I just end up with \(-\frac{b}{2a}\).
And that, that’s helpful to us as well because it will tell us where the axis of symmetry, it’ll tell us where the vertex is and on the parabola. The vertex tells us either a maximum or a minimum. (Figure 5)

So in this particular case, let’s walk through it. Negative b right there, negative and \(-1\) squared and then \(-4a\), and there’s “c” and I think if I’ve done my arithmetic right, then that becomes a one. This would be a one plus 20. So it should be \(\frac{1-\sqrt{20}}{2}\) and \(\frac{1+\sqrt{20}}{2}\), which works out to be these two answers which I alluded to just kind of inside the two previous answers, okay? So that looks plausible. I think that’s going to work. And it even tells me that the axis of symmetry is at \(x = \frac{1}{2}\). Negative negative one is one, two times one is two. So there’s my axis of symmetry and it locates the vertex for me. So that’s useful in doing max and min type problems. (Figure 6)

![Figure 6, Segment 3](image)

I feel like I ought to talk about at least one problem, so let’s look at this one. We don’t have a world trade center any more, it was destroyed in a terrorist attack. But that was a huge building and I remember when that attack occurred, there were stories about the debris falling and even people jumping out of the building to avoid the heat from the flames and things like that. And I remember thinking, “Wow, how long a fall is that?” I mean that’s a huge fall. You know, it’s a quarter mile and I have no concept of that myself as to how far, how long you would . . . ? Does it take three seconds to hit the ground? Does it take 30 seconds to hit the ground? I don’t know. So let’s do the arithmetic. (Figure 7)

The equation that governs this and I don’t have time to derive this, is \(-16t^2\) plus initial velocity up plus the initial height. And I’m doing everything here in feet and seconds. So
when I go through, I’ve got the $-16t^2$. The height here, the very tip top of this would have been one thousand, three hundred 53 feet, and to get the answer, I just need to solve that for zero, which I plug in the quadratic formula. Zero here for the “b”, the coefficient for the “b” term is zero. And it turns out to be a plus and minus this over negative 32. One answer is negative, I don’t want that answer. The positive answer is about 9.2 seconds. So that’s a long time. Now the one thing I’m not doing here because it’s more difficult is I’m not taking into account the wind resistance which would slow you down. But I have on the other hand calculated the fall from the very tip top of the thing which would make for a longer trip. So really when you kind of do the give and take, that was probably standard and that’s time to think, you know that’s kind of scary. (Figure 8)

I’ve got some summary pages here that I’ll of flash up that kind of show what we’ve been talking about.

Closing

Hope you’ve enjoyed the show today. I tried to show you some cool examples and reasons that you need to know these things, but I’m always looking for other examples. So in a few seconds here, we’re going to flash up the web page where you can find these, all of the episodes and computer files that you could download. You’ll also find an email address there that you contact me. Please do that, give me some good ideas. With that, I am out of here. Thank you for your time, I’ll see you next week.
Example The World Trade Center was 1,353 feet tall. How long would it take an object to hit the ground from that height, neglecting wind resistance?

Solve $-16t^2 + 1353 = 0$ for $t$.

$$t = \frac{0 \pm \sqrt{0^2 - 4(-16)(1353)}}{2(-16)} = \pm\sqrt{86592}$$

$$t \approx 9.2 \text{ seconds}$$

Summary Quadratic equations can have 0, 1, or 2 real solutions.

$$ax^2 + bx + c = 0$$

no real solution 1 real solution 2 real solutions
Summary

All solutions (real and complex) of the quadratic equation $ax^2+bx+c = 0$ can be found using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Quadratic equations occur in situations involving gravity, area, volume where one length measurement is known, etc.