1 Consumer mathematics – Paying off credit card debt

Objective: To show how a better understanding of the mathematics of calculating debt payments can influence our decisions in financial matters. Specifically, we examine credit card debt, and the time necessary to pay-off a “max-ed out” card. The method for doing that is equivalent to the mathematics for calculating the payment on an amortized loan.

Hello. Welcome to “Math Matters: Why do I need to know this?” The object of our show is to take the entry-level mathematics that we teach in our general math, college algebra, and teacher-ed courses and show you how it applies to everyday life and I’ve got some nice examples to show today. There’s a little bit of algebra I am going to have to kick around today. And I will take you right to the edge with what you can do with algebra before I let go of you today.

I want to start with something that is probably important to most college kids. I want to take a look at credit card debt and how to calculate how to get out of debt on these credit cards. To tell you the truth, and you may have figured this out, credit card companies will almost give you a credit card, without much background check or anything. They love giving credit cards to college kids, because you tend to, college kids tend to do things on the irresponsible side. It’s the old buy now, pay later kind of plan. And you can get yourself into trouble doing things like that. My little catchy slogan – BUY NOW! PAY LATER! – a lot of kids have gotten themselves into trouble with that strategy. And they end up paying for that years and years and years. I know adult folks my age who are still paying off credit cards that they have maxed out when they were younger. I don’t want to be completely anti-credit card – they can be a good thing. They’re quite helpful for establishing credit. If you are traveling anywhere, it’s probably not a bad idea to have a credit card so you can reserve a hotel room, things like that. And it’s very convenient so I’m not being anti-credit. I’m just saying they will kind of focus on college kids. They have actually started limiting, you know, they used to set up stands around here and give out t-shirts and things all over the place and now they have stated cutting back on this because kids were getting into trouble.

Let me introduce some terminology that, I will say these words over and over, so I want to make sure you know what I am talking about. Annual percentage rate is the interest you are going to pay on borrowed money on this card and that’s a little bit of a gimmicky thing too. A lot of cards, a lot of banks will try to get you to use their cards based on a low introductory APR but that’s going to balloon up later. You’d better keep your eye on that and read the fine print when you are looking at things like that. Most credit cards use the average daily balance method for calculating interest that you are going to have to pay. What you do to get this, we may do another example like this later, is they take a look at the balances at the end of each day of the pay cycle and they average them up. And they use that as the amount
you owe kind of consistently throughout the month. And then finance charge – that’s the interest you are going to pay on a balance. And if you are paying off the card each time, you won’t have to pay this finance charge, but if you carry a balance on a card, you will have to pay the interest on what you owe. And that’s not actually a bad calculation – it’s principal times rate times time. Really, it’s the average daily balance times the APR then divide by 12 because you are doing it monthly. You are going to do a monthly calculation. (Figure 1)

**Terminology:**

**Annual Percentage Rate (APR)** – annual interest rate on borrowed money.
(Beware low introductory APR’s that shoot up later!)

**Average Daily Balance (ADB)** – average of the balances at the end of each day of the pay cycle.

**Finance Charge** – interest on balance, only paid when you carry a balance.

\[ = ADB \times \frac{APR}{12} \]

(Figure 1, Segment 1)

All right, so if you build up a large balance – a lot of people have found themselves in this position – how long will it take to pay it off? And I am going to make a few assumptions here so we can calculate how to do this. Let’s say your balance is \( B_0 \), the initial balance, and the APR I will just refer to as \( r \). Let’s say, let’s make things as simple as possible. Let’s say we are going to stop using the card and we are going to pay \( x \) per month until it’s paid off, okay? So, basically, the card is no longer useful to us as a purchasing item. We are just trying to get the darn thing paid off. That makes the average daily balance business easier because if I’m not changing the balance everyday, then that balance is just the average daily balance. We will just use that \( B_0 \) as the average daily balance. And I’m going to simplify things a little bit by letting \( m \) be the monthly rate instead of having to write this fraction \( r \) divided by 12 all the time – I will just use an \( m \), okay? (Figure 2)

So after the first month with this balance you’ve got the hundred percent of what you started with, plus the monthly interest you are going to have to pay, but you are going to pay part of it off – that is this \( x \). And then when you come back and do this for the second month, this \( B_1 \), this is the new balance. And it’s your new average daily balance for that month. It’s the same calculation if you take this and shoot it right in there. Looks kind of messy, but
you can clean it up by kind of distributing this among the terms and you are getting a nice pattern here see the square here for the first power, the zero power. If you do it again, it’s a little messy but I’m going to shoot this whole thing right into there. And so this times this times this times this, and you see the pattern I have developed there, you can factor this x out and you have a cool little pattern going on there. So in general, the balance left after n months of making payments is this messy looking thing. (Figure 3)

Now I’m going to clean this up. This is a certain amount raised to a power plus raised to one less power all the way to zero power. And I am just going to call this p for ease of notation there. So that is what I am trying to add up. There is a cool way to do this, there is a nice little formula. That would be S, whatever it is, and multiple through by p and then subtract. Your one becomes a negative p, this one becomes a negative p squared, and so forth, and you have all these terms that cancel out. Can you see that? The p and the minus p and so forth. So when you subtract, you get one minus p times the S, and then the one here and minus the p in there, and then just divide through by the junk. So you get this cool formula here that you can replace this business with. And it looks like that. (Figure 4)

Now I know, I know that’s messy. But here is what we are trying to do – we are trying to make that last balance in the nth month zero. So set that equal to zero, and then solve for the x. Well, move all this stuff to the other side. And then you multiple by the reciprocal of this fraction so the m comes up here and this business comes to the denominator. And that is the amount you need to pay off the balance in n months. This is actually a pretty useful formula if you are doing things like amortized loans and so forth. You would use the same basic formula. That’s where you are paying a fixed amount that doesn’t change throughout the

**Question:** If you do build-up a large balance, how long will it take to pay it off?
Let the balance = B₀ and the APR = r.
We will stop using the card and we will pay x per month until it is paid off.

Since we have stopped using the card, then the beginning ADB will equal B₀.

Let \( m = \frac{r}{12} \) be the monthly rate.
1st: \( B_1 = B_0(1 + m) - x \)

2nd: \( B_2 = B_1(1 + m) - x \)
\[ = \left( B_0(1 + m) - x \right)(1 + m) - x \]
\[ = B_0(1 + m)^2 - x(1 + m) - x \]

3rd: \( B_3 = B_2(1 + m) - x \)
\[ = \left( B_0(1 + m)^2 - x(1 + m) - x \right)(1 + m) - x \]
\[ = B_0(1 + m)^3 - x(1 + m)^2 - x(1 + m) - x \]
\[ = B_0(1 + m)^3 - x\left( (1 + m)^2 + (1 + m) + 1 \right) \]

---

**Figure 3, Segment 1**

---

In general, the balance left after \( n \) months of making payments of \( x \) is

\[ B_n = B_0(1 + m)^n - x\left( \sum_{p=1}^{n} (1 + m)^{n-p} \right) + \cdots + (1 + m) + 1 \]

\[ S = 1 + p + \cdots + p^{n-2} + p^{n-1} = ? \]
\[ -pS = -p - p^2 - \cdots - p^{n-2} - p^{n-1} - p^n \]

\[ (1 - p)S = 1 - p^n \]

Then
\[ S = \frac{p^n - 1}{p - 1} \]

\[ B_n = B_0(1 + m)^n - x\left( \frac{(1 + m)^n - 1}{m} \right) \]

---

**Figure 4, Segment 1**
month, but then you are making a set payment too. So we may come back and do amortized loans with it. (Figure 5)

\[
\begin{align*}
\text{Now set } B_n &= 0: \\
B_0(1+m)^n - x \frac{(1+m)^n - 1}{m} &= 0 \\
B_0(1+m)^n &= x \frac{(1+m)^n - 1}{m} \\
x &= \frac{B_0m(1+m)^n}{(1+m)^n - 1}
\end{align*}
\]

is the amount needed to pay off the balance in \( n \) months.

Figure 5, Segment 1

So you have a 4,000 dollars balance on a credit card with a 14.15% interest APR and that’s pretty standard. In 2 years or 24 months, you would have to pay $167 just to pay back, you know, that is 1/24th of what you owe. And then if you go through and do this calculation with \( n \) equal to 24, it works out to be a $192.34. And if you multiply that by 24, you realize you are paying about $616 dollars worth of interest over two years that you wouldn’t normally have to pay. (Figure 6) If you want to drag that out over five years – now see if you catch the moral here – then your are paying 2/3 of a hundred dollars of what you owe. And then the interest, you are paying $93.38 a month. You multiply that by 60, and you are paying back over $1,600 of just interest on the money that you borrowed. So it can really be an expensive proposition you need to be aware of when you decide to use a credit card. (Figure 7)

That was a lot of algebra really quick so let me flash those things up on a summary page for you to kind of digest.
Suppose you have a $4,000 balance on a credit card with APR 14.15%.
To pay off in 2 years (24 months,)
\[
\frac{4000}{24} \approx 166.67 \quad m = \frac{0.1415}{12} \approx 0.01179167
\]
\[
x = \frac{B_0 m (1 + m)^n}{(1 + m)^n - 1} = \frac{4000m(1 + m)^{24}}{(1 + m)^{24} - 1} \approx 192.34
\]
Notice that 24($192.34) = $4616.16, so it costs you $616.16 worth of interest.

Figure 6, Segment 1

Suppose you have a $4,000 balance on a credit card with APR 14.15%.
To pay off in 5 years (60 months,)
\[
\frac{4000}{60} \approx 66.67 \quad m = \frac{0.1415}{12} \approx 0.01179167
\]
\[
x = \frac{B_0 m (1 + m)^n}{(1 + m)^n - 1} = \frac{4000m(1 + m)^{60}}{(1 + m)^{60} - 1} \approx 93.38
\]
Notice that 60($93.38) = $5602.80, so it costs you $1602.80 worth of interest.

Figure 7, Segment 1
Summary

**Annual Percentage Rate (APR)** – annual interest rate on borrowed money. (Beware low introductory APR’s that shoot up later!)

**Average Daily Balance (ADB)** – average of the balances at the end of each day of the pay cycle.

**Finance Charge** – interest on balance, only paid when you carry a balance.

---

Summary page 1, Segment 1

---

Summary

\[ 1 + p + \cdots + p^{n-2} + p^{n-1} = \frac{p^n - 1}{p - 1} \]

In order to pay off a non-used credit card, with balance \( B_0 \) and an APR of \( r \), in \( n \) months, one will have to pay

\[ x = \frac{B_0 m (1 + m)^n}{(1 + m)^n - 1} \]

per month,

where \( m = r/12 \) is the monthly rate.

---

Summary page 2, Segment 1

---
2 Circles and Pythagorean theorem – Finding yourself on a map

Objective: To illustrate the relevance of knowing the equation of a circle and equation-solving techniques by showing how it could impact the outcome of a potentially life-threatening real-world situation. We will develop the equation of the circle using the Pythagorean theorem.

I want to take you through a bit of algebra today that involves the equation for a circle, and at the same time it’s going to involve Pythagorean theorem. That is one of the things that we teach in our algebra courses. The kids are kind of like, “Why do I need to know this, I’m never going to use this, ever.” Okay, well you just don’t know that. You just can’t be sure of that. I want to give you a situation today where it might be helpful, could even save your life or something like that.

To make sure you know what I am talking about let me introduce the Pythagorean theorem. Most people are familiar with the Pythagorean theorem. If you look at a right triangle – let’s flash this picture up – if look at a right triangle, and we say the length of the sides, called the legs, are a and b and this side opposite the right angle is called the hypotenuse. If that is length c, then the Pythagorean theorem is pretty simple, it’s the sum of the squares of the legs and that equals then the square of the length of the hypotenuse. Most people have seen that. But, when we get to circles people get really confused. If we are looking at a circle and it has coordinates – I’m thinking of this on a coordinate axis – the center of the circle has coordinates \((h, k)\), then, if I think about a general point on that line outside – I didn’t put a point there because it could be any on the outside of that circle – we can actually draw a right triangle inside of this. If I go from the center to the x coordinate and then up to the point that’s a right triangle. And the length of this side is \(x - h\), or \(h - x\). You know, there could be a sign here. It doesn’t really matter. And the length of this side is \(y - k\). And the circle radius is \(r\). So this is your \(a\), this is your \(b\), and this is your \(c\) – Pythagorean theorem. There’s the equation of the circle. (Figure 1)

Now I want to describe a situation – kind of hold unto this if you can, maybe you have seen that before – I want to describe a situation where knowing that could actually be helpful to you. Let’s say you are hiking through the woods and you’ve got some supplies with you, but don’t really intend on being injured in the woods. You twist an ankle or something like that and you just can’t get home, you can’t walk on it. So you’ve got your map, you’ve got your cell phone, and you know you’ve got a rangefinder so you can tell how far away things are, but you didn’t bring safety items like a GPS tracker or flares, or, you know, anything crazy like that. But you can, from where you are sitting or injured, you can see two ranger towers, two things sitting off in the distance. One’s kind of to the southeast and one’s kind of to the southwest. So using your rangefinder you can estimate I think I have 1,200 yards from the southwest tower and I’m about 900 yards from the southeast tower. (Figure 2)

Now that’s enough information that you should be able to use your cell phone and give your position to emergency personnel. The quickest and the thing you might want to do first is the thing you can do is to simply draw circles of a particular radius and this is where it gets kind of into estimating things. Here’s the map that you have. And you know you’ve got
Situation You are hiking through the woods when you severely turn your ankle, and can not hobble home. You have a map, a cell phone, and a range-finder, but no GPS tracker or flares. You see two ranger towers, one to the southeast and one to the southwest. You estimate that you are 1200 yards from the southwest tower and 900 yards from the southeast tower. Give your position to emergency personnel.
a scale down there somewhere. And chances are you don’t have a compass with you. And I don’t mean a compass that tells you north, south, east, and west. I mean a compass that we use to draw circles. If you didn’t bring a GPS thing, then you certainly didn’t bring your compass along, but you could use a piece of string or something to estimate and you could draw the circle with a radius of 1200 from the southwest tower or just that arc – you don’t have to draw the whole circle. And then draw the arc of the circle that’s 900 yards away from the southeast tower. And then where these things cross is right here. That is your position. I mean based on your estimations. So you could immediately call up and say I’m about 900 yards away east of this tower here you know generally which one you were at, and then about, I don’t know 800 or so yards north of that. So you could kind of give a loose estimate of your position to them. (Figure 3)

![Figure 3, Segment 2](image)

If you have a compass, you can draw arcs of the correct radius.

Now, if you have done that, then you want to sit there and so, you know, while you are waiting you might want to go head and calculate your exact location. Well, you can do that using algebra, because each of those circles you just drew has an equation and what you are looking for here is the intersection of the two circles. So you are looking for an $x$ and a $y$ that satisfy the equations of both circles at the same time, okay? So this is kind of the algebra method and I am going to recommend that we switch to miles because of the numbers I had my map in yards, but if you divide by 1760, which is the number of yards in a mile, then the numbers do get a little smaller, some fractions, but not too bad. So this is the circle of radius 1200 and this is the circle, notice that it has a center at one mile $x$ and then the $y$ coordinate was a quarter mile up and then that is 900 yards converted to miles. (Figure 4)

So you can solve for $y$ by solving for $y^2$ first and then taking the square root of both
sides. That generates a plus and a minus, but we’re looking down, we are looking to the south for these things, so I want the upper part of here, I want the plus part of this, okay? The other one you are going to have to multiply things out. I know it’s messy but you can combine some terms and you can actually, . . . hang on let me get a little room to work here. (Figure 4) Actually, you can replace the $y^2$ with this formula for $y^2$ in terms of $x$. So the $x^2$ then disappears, and you can add this and this, kind of getting those things combined. And all of a sudden your squares are all gone. I mean, you can know solve, for example, $y$ in terms of $x$, by just moving this to the other side, multiplying everything but two, and so there is a nice formula for $y$. (Figure 5)

And then to solve for $x$ we can just jam this back into one of these two equations. I will put it back into the simplest of the two. And then multiply things out. I going to be honest with you. I got tired of dealing with all these fractions and so I just went to numerical things, and that’s probably what you would do, too. I know you are saying “You don’t pack calculators, doofus, so how are you suppose to do this?” Well, most of you have a cell phone right? That has a calculator on it. This is one of the oldest, most archaic cell phones going probably – most of them now have cameras and calculators and web stuff and I can just talk on my phone. I’m really just behind the times. But then you can use your calculator to do all these decimals things and then use the quadratic formula just to solve for $x$. The $x$ I want is actually this one. If you remember the map, there are two points where these two can cross, one of them was south, one of them was north of the towers. And this is the one that corresponds with the north point so I will get rid of that. Converted back into yards that is about 922 yards to the east of that first tower. If you plug this back into the formula up
Here, that gives you your y coordinate in miles and then you convert that back into yards. (Figure 6) And so sure enough, if you go check this coordinate you are about 768 yards north of that other tower. That is that point right there. (Figure 7)

So you can give pinpoint directions simply by using algebra and not have to have one of those GPS things, or all the expensive kind of things you can purchase to save your life. While they are scrambling to this general area right in here, you can say “I figured it out. I’m 921 yards east of the southwest tower and 768 yards north of that.” And they should be able to come right to here.

A little algebra trick there, hopefully it could save your life one day. They keep kidding about how this that this stuff could save your life one day, and here’s an episode where it really could. I’m going to flash up some summary pages of the last things that we did, and we will come back for another example.
Substituting back into the first circle,
\[ x^2 + \left( \frac{9803}{3872} - 4x \right)^2 = \frac{225}{484} \]
\[ 17x^2 - 20.2541x + 5.94497 = 0 \quad \text{(approx.)} \]
Using the quadratic formula,
\[ x \approx 0.523833 \text{ or } 0.667587 \text{ miles} \]
\[ x \approx 921.946 \text{ yards} \]
\[ y \approx \frac{9803}{3872} - 4(0.523833) \approx 0.436434 \text{ miles} \]
\[ y \approx 768.125 \text{ yards} \]

Figure 6, Segment 2

\[ \approx (922,768) \]

Southwest tower

Southeast tower

Figure 7, Segment 2
Summary

**Pythagorean Theorem**

\[ a^2 + b^2 = c^2 \]

Summary page 1, Segment 2

---

Summary

**Circles**

\[(x - h)^2 + (y - k)^2 = r^2\]

Summary page 2, Segment 2
3 Exponential and logarithmic functions – Solving the boiling-water myth

Objective: To illustrate how mathematical techniques can be as or more effective as experimentation in proving or disproving physical phenomena. In particular, we model the time it takes water to boil based upon its initial temperature, using a model that derives from Newton’s Law of Cooling. We are careful to use only algebra in our work, but we show the limitations of algebra in this case and foreshadow the usefulness of calculus in this type of problem.

This is about the time during the semester where we start to cover exponential functions or logarithm functions and we do, in the course of doing that, talk about some applications, but I want to show you a neat application of both types of functions. And also, when you are learning algebra it’s good to know what you can do with it, and what you can’t do with it. I think it is good to learn the limitations of what you are doing. And then, if you are very interested, you know where to go, I mean, you can keep going on to more complicated things.

One of my favorite shows, and I don’t watch a lot of TV, but I watch this show and Mythbusters on the Discovery Channel, and I love these guys. I mean, these are some really talented guys. They take all these myths they have picked up on urban legends and things, and they set about trying to prove or disprove them by using elaborate experimentation. I mean, they have manufactured some magnificent things – chicken cannons and marching robots and all these kinds of things. I love these guys. I am going to do the same thing with you today, although I’m not going to build the elaborate stuff. I mean, I don’t have the resources or the talent. But I do know math, so I’m going to do my mythbusting with mathematics. I am going to use a little, mostly algebra, but I will end up using a little bit of calculus, but it’s not going to happen in front of you. I’m simply going to mention that to get this, really, you’d have to use calculus, so that you kind of know where you can go with the algebra and where you have to go to get more. Okay?

Here is the myth I want to look at today, And this is one . . . , I remember my grandmother telling me this. She swore up and down that this was true: a pot of ice water would boil faster than a pot of warm water. If you really wanted it to happen faster you would get a pot of ice water and it would boil faster than warm water. The notion behind this, it is actually based on Newton’s Law of Cooling, although I don’t think she realized that. And that is, Newton’s Law of Cooling says that the rate of the change of temperature of something is proportional to the difference in the temperature of the thing and the surrounding medium. And that is certainly true. I am not going to try to bust Newton’s Law of Cooling; it is certainly true. That is something like if you take something (water) and you apply something (a lot of heat) to it, starting out, the temperature will change dramatically. And then, as it gets close to the temperature of the heat you are applying, it will slow down. (Figure 1)

So I’m not saying, okay, I’m not saying Newton’s Law is incorrect. In fact it is very correct, but there are two things working against working against each other here. This business of the rate of change slowing down, okay, and being faster when the difference is great, and then also if the temperature is lower it has to heat up more, so it makes sense that if would take more time. So which one kind of wins out?
Well, here’s just to kind of show you I believe in Newton’s Law of Cooling and heating, actually, whatever. I conducted an experiment. I put a pot of room temperature water on the stove. I had a thermometer that I kind of put in that, and I measured the temperature. I turned the heat on, and measured the temperature every 30 seconds until it started to boil. And here is where I plotted the points, the horizontal line there is 212 degrees, which is boiling point of water. Water will not get higher than that; it stops heating up at that point. It took exactly nine minutes to do this. (Figure 2)

Now, using calculus, I can prove that the model for this kind of growth this is not a linear model, although it’s almost a line. You can detect a slight curve there. The model would follow this kind of format. The initial temperature minus the temperature of the surrounding medium, okay, times $e^{kt}$. We have talked about this before: $e$ is this marvelous, irrational number that we use a lot. The $k$ is a constant that is kind of dependent on the substance that you are heating. And then this again is the temperature of the medium or the heat that you are applying, okay? So I went through and actually solved for the things in my graph. The initial temperature was 68 degrees Fahrenheit as it works out. Apparently the heat that I applied to this water was 394 degree Fahrenheit and the $k$ that goes along with my substance water is right here. So there is the curve that kind of matches up. (Figure 2)

Solving for the above model does require calculus – we have to solve the differential equation given by Newton’s Law of Cooling,

$$\frac{dT}{dt} = k(T - T_{M})$$

with initial condition $T(0) = T_0$. 

---

**Myth?** A pot of ice water will boil faster than a pot of warm water.

This “myth” is based on **Newton’s Law of Cooling**

The rate of change of temperature of a substance is proportional to the difference in temperature of the substance and the surrounding medium.

$$\text{rate} = k(T - T_M)$$
The differential equation is separable, so
\[
\int \frac{dT}{T - T_M} = \int k \, dt + C,
\]
\[
\ln |T - T_M| = kt + C, \text{ and}
\]
\[
|T - T_M| = e^{kt+c} = e^C e^{kt}.
\]
We may drop the absolute values by letting \( K = \text{sign}(T - T_M)e^C \), where
\[
\text{sign}(x) = \begin{cases} 
1 & \text{if } x > 0 \\
0 & \text{if } x = 0 \\
-1 & \text{if } x < 0.
\end{cases}
\]
Then
\[
T(t) = Ke^{kt} + T_M.
\]
Letting \( t = 0 \) and using our initial condition to solve for \( K \), we have \( K = T_0 - T_M \). Thus, our model has the form used in Figure 2.

As far as finding the curve that best matches my data \( \{ (\frac{n}{2}, y_n) \}_{n=0}^{18} \), I measured the initial temperature \( T_0 \) to be 68° F with my thermometer, and I used Mathematica to find \( k \) and \( T_M \) that numerically minimize the sum
\[
\sum_{n=0}^{18} \left( y_n - T \left( \frac{n}{2} \right) \right)^2.
\]
Now what I would like to do, I would like to prove one way or another that this myth is false. So I’m going to let the temperature of the medium be constant. Whatever that is I will call \( C \). I am just going to make sure it is more than the initial temperature, so that \( k \) would have to be negative and I am going to set the temperature to 212. So then I am going to solve for time in terms of the initial temperature \( t_0 \). So the blue things here are constants. I am going to treat them as constants. I want to solve for \( t \) in terms of \( t_0 \); \( t_0 \) will be my independent variable. And \( t \) will be my dependent variable. (Figure 3)

Conclusive proof Let \( T_M \) and \( k \) be constant \( (T_M = C > T_o, k < 0, ) \) and set the temperature \( T = 212 \). Solve for the time \( t \) in terms of the initial temperature \( T_0 \).

\[
(T_0 - C)e^{kt} + C = 212
\]

If the “myth” is true, then the graph of \( t \) must have a “low place” in it. (Figure 3)

Okay, here is what I am looking for – if I graph this thing once I solve for \( t \), I put \( t_0 \) on the horizontal axis and \( t \) on the vertical axis. If my graph looks something like this, then it has a low place in it. This would be a point that if I started with this initial temperature, it would take less time than a higher initial temperature would take. I am looking for a little swag like them – a little dipzy-do, okay? (Figure 3)

Alright, so let me take this thing, and let’s set about solving for \( t \). I would bring, it’s just a little bit of algebra here, I would just bring \( C \) over to the other side. Divide by the junk here, okay? And to solve for an exponent like this you would have to take the log of both sides, and it’s convenient to take the log base \( e \), which is the natural log. Oh, and I should mention that I can turn these around, and I can take a negative out of the top and the bottom. This is bigger than this, and this is less than one. So when I do take the natural log of both sides, this is a negative number, okay? But if \( k \) is negative, then when I divide by the \( k \), I get a positive amount. So I have a positive amount – that makes sense. And also if \( I \), if I get my initial temperature up to 212 and then if I measure how long it will take me to get to 212, well, no time at all. It will be right there. But that doesn’t give me a good tip as
to whether it ever levels off or does that little dipzy-doodle thing. (Figure 4)

Well, algebra-wise, all I can do is look at examples. So I will look at the example from my experiment. Here is my initial, I mean this is the temperature of my medium. And then this is the $k$ that I had. And when I graph this curve there are no low places there, right? It’s strictly decreasing. And it would certainly be false in that case. Ah, this is where you have to go past algebra. You can actually use calculus to show that that curve never levels off. Then, if it never levels off, it is never going to have that low place in it. So, I declare officially and conclusively that this myth is busted. There is no way, no way that could ever happen. My granny, I think, was wrong, bless her heart. She was right about a lot of other good things, but not this one. (Figure 5)

In order to prove this for more than just my pot of water on my stove, we need to look at the general model

$$
t(T_0) = \frac{1}{k} \ln \frac{C - 212}{C - T_0}, \quad C > T_0,
$$

and set the derivative of $t$ with respect to $T_0$ to zero. (In order to get the dipzy-doodle, I need at the very least one place with a horizontal tangent line.) Using the properties of logarithms, we know that

$$
t(T_0) = \frac{1}{k} \left[ \ln(C - 212) - \ln(C - T_0) \right],
$$

so

$$
t'(T_0) = \frac{1}{k(C - T_0)} \neq 0.
$$
Thus, the curve has no dip, and the myth is busted.

*I ran through some stuff pretty quick there, so let me flash up some summary pages and we will be right back.*

**Closing**

*Well, I hope you have enjoyed the examples I have showed today. I was looking for some cool stuff, some applicable stuff in the kind of things that we cover in math in the entrylevel courses. I am always looking for your input, so if you have some cool things that you have covered in your math class, or simply things that you have encountered and you find simply useless, please let me know and I will take the challenge on that. Otherwise I am done. Hope to see you next week.*
Summary

**Newton’s Law of Cooling**
The rate of change of temperature of a substance is proportional to the difference in temperature of the substance and the surrounding medium.
rate = \( k(T - T_M) \)

Calculus gives us the model
\[
T(t) = (T_0 - T_M)e^{kt} + T_M.
\]

Summary page 1, Segment 3

---

Summary

To solve for a variable in the exponent of an exponential function, we usually take the log of both sides.

**Myth** A pot of ice water will boil faster than a pot of warm water.

**BUSTED!**

Summary page 2, Segment 3