Preface

This book presents ordinary differential equations (ODE) and partial differential equations (PDE) under one cover. All topics that form the core of a modern undergraduate and the beginner's graduate course in differential equations are presented at full length. We especially strived for clarity of presenting concepts and the simplicity and transparency of the text. At the same time, we tried to keep all rigor of mathematics (but without the emphasis on proofs—some simpler theorems are proved, but for more sophisticated ones we discuss only the key steps of the proofs). In our best judgment, a balanced presentation has been achieved, which is as informative as possible at this level, and introduces and practices all necessary problem-solving skills, yet is concise and friendly to a reader. A part of the philosophy of the book is "teaching-by-examples" and thus we provide numerous carefully chosen examples that guide step-by-step learning of concepts and techniques. The level of presentation and the book structure allows its use in engineering, physics, and mathematics departments.

The primary motivation for writing this textbook is that, to our knowledge, there has not been published a comprehensive textbook that covers both ODE and PDE. A professor who teaches ODE using this book can use the PDE sections to complement the main ODE course. Professors teaching PDE very often face the situation when students, despite having an ODE prerequisite, do not remember the techniques for solving ODE and thus can't do well in the PDE course. A professor can choose the key ODE sections, quickly review them in the course of, say, three or four lectures, and then seamlessly turn to the main subject, i.e., PDE.

The ODE part of the book contains topics that can be omitted (fully or partially) from the basic undergraduate course, such as the integral equations, the Laplace transforms, and the boundary value problems. For the undergraduate PDE course the most technical sections (for instance, where the nonhomogeneous boundary conditions are discussed) can be omitted from lectures and instead studied with the accompanying software. At least Sections 1 through 7 from Chapter 8, Fourier Series, should be covered prior to teaching PDEs. For class time savings, a few of these sections can be studied using the software.

The software is a very special component of the textbook. Our software covers both fields. The ODE part of the software is fairly straightforward—i.e., the software allows readers to compare their analytical solution and the results of a computation. For PDE the software also demonstrates the sequence of all the steps needed to solve the problem. Thus it leads a user in the process of solving the problem, rather than informs of the result of solving the problem. This feature is completely or partially absent from all software that we have seen and tested. After the software solution of the problem, a deeper investigation is offered, such as the study of the dependence of the solution on the parameters, the accuracy of the solution, the speed of a series convergence, and related questions. Thus the software is a platform for learning and investigating all textbook topics, an inherent part of the learning experience rather than an interesting auxiliary. The software enables lectures, recitations, and homeworks to be research-like, i.e., to be naturally investigative, thus hopefully increasing the student rate of success. It allows students to study a *limitless* number of problems (a known drawback of a typical PDE course is that, due to time constraints, students are limited to practicing solutions of a small number of simple problems using the “pen and paper” method).

The software is very intuitive and simple to use, and it does not require students to learn a (quasi)programming language as do the computer algebra systems (CAS), such as Mathematica and Maple. Most CAS require a significant time investment in learning commands, conventions, and other features, and often the undergraduate students are very reluctant, especially if they have reasons to think that they will not use CAS in the future; furthermore, the instructors are often not willing to spend valuable classroom time teaching the basics of using CAS. Besides, where using CAS to solve an ODE is a matter of typing in one command (*dsolve* in Maple or *DSolve* in Mathematica), which students usually can learn how to do with a minor effort, solving a PDE in CAS is more complicated and puts the burden on the instructor to create a worksheet with several commands, often as many as ten or fifteen, where students just change the arguments to enable the solution of an assigned problem. Creation of a collection of such worksheets that covers all sections of the textbook is only possible when the instructor teaches the course multiple times.

The software and tutorials contain a few topics, such as the classical orthogonal polynomials, generalized Legendre functions, and others, which are not included in the book to avoid its overload with content that is presently rarely taught in PDE courses (at least in the U.S. academic system). These topics with the help of the software can be assigned for an independent study, essay, etc.

The software tutorials for different chapters are placed in the appendices.

Finally, we would like to suggest the book *Mathematical Methods in Physics* [1] as a more complete and advanced PDE textbook. That book is written in the same style and uses the previous version of the software.

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