

MATH 117

Solving for Angles

We can use the *inverse trig functions* on a calculator to solve for the angles that give a particular trig function value. In other words, given a specific value v , we can find the angles θ such that

$$\sin \theta = v \quad \text{or} \quad \cos \theta = v \quad \text{or} \quad \tan \theta = v$$

We always take the angles to be between 0° and 360° , and in each case, there will be *two* solutions. However, the calculator will only give *one* angle, so we will need to find the appropriate symmetric angles in the correct quadrants to give the solutions.

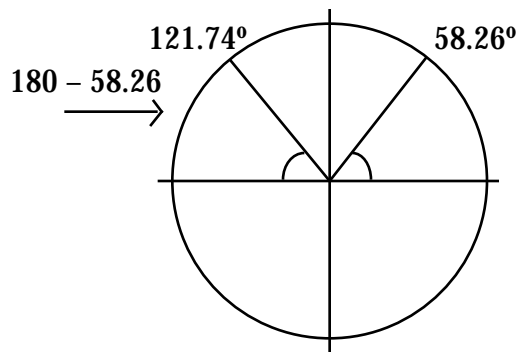
Example. To the nearest 100th, find the angles between 0° and 360° such that

(a) $\sin \theta = 0.850444$	(c) $\cos \theta = 0.91706$	(e) $\tan \theta = 0.18101$
(b) $\sin \theta = -0.916921$	(d) $\cos \theta = -0.78801$	(f) $\tan \theta = -1.5$

Solution. (a) There are two angles where $\sin \theta = +0.850444$, one in the 1st Quadrant and one in the 2nd Quadrant.

The command $\sin^{-1}(0.850444)$ gives $\theta = 58.26^\circ$.

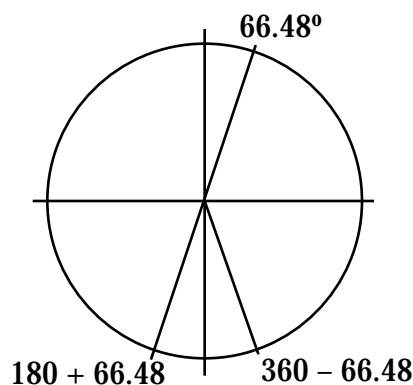
The corresponding angle in the 2nd Quadrant is $180^\circ - 58.26^\circ = 121.74^\circ$.



(b) There are two angles where $\sin \theta = -0.916921$, one in the 3rd Quadrant and one in the 4th Quadrant. Initially, we can ignore the “negative” to find the reference angle in the 1st Quadrant: $\sin^{-1}(0.916921)$ gives $\theta = 66.48^\circ$.

The corresponding angles in the 3rd and 4th Quadrants are $180^\circ + 66.48^\circ = 246.48^\circ$ and $360^\circ - 66.48^\circ = 293.52^\circ$.

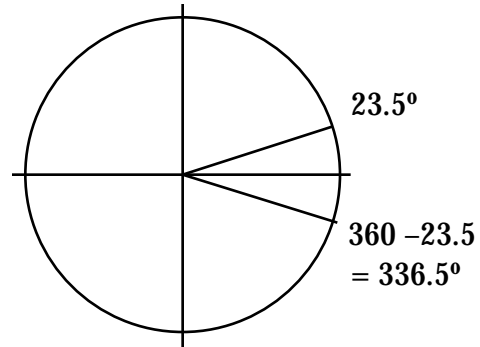
Thus, the solutions are 246.48° and 293.52° .



(c) There are two angles where $\cos\theta = +0.91706$, one in the 1st Quadrant and one in the 4th Quadrant.

The command $\cos^{-1}(0.91706)$ gives $\theta = 23.5^\circ$. The corresponding angle in the 4th Quadrant is $360^\circ - 23.5^\circ = 336.5^\circ$.

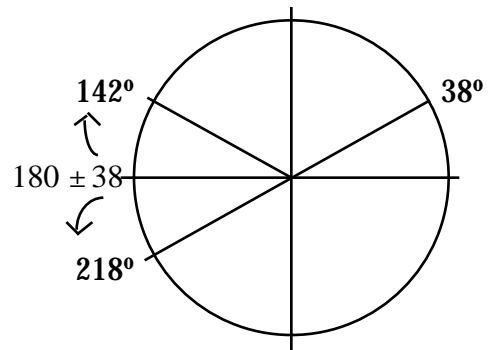
Thus, the solutions are 23.5° and 336.5° .



(d) There are two angles where $\cos\theta = -0.78801$, one in the 2nd Quadrant and one in the 3rd Quadrant. Initially, we ignore the “negative” to find the reference angle in the 1st Quadrant: $\cos^{-1}(0.78801)$ gives $\theta = 38^\circ$.

The corresponding angles in the 2nd and 3rd Quadrants are $180^\circ \pm 38^\circ$, which give 142° and 218° .

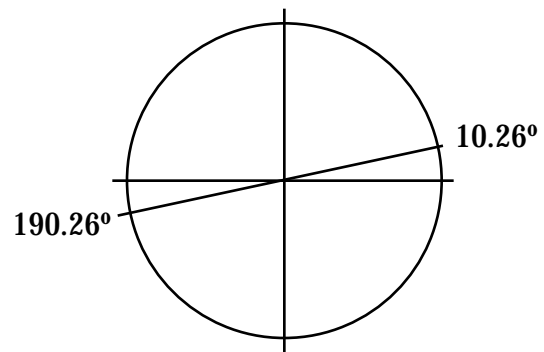
Thus, the solutions are 142° and 218° .



(e) There are two angles where $\tan\theta = +0.18101$, one in the 1st Quadrant and one in the 3rd Quadrant.

The command $\tan^{-1}(0.18101)$ gives $\theta = 10.26^\circ$.

The corresponding angle in the 3rd Quadrant is $180^\circ + 10.26^\circ = 190.26^\circ$.



(f) There are two angles where $\tan\theta = -1.5$, one in the 2nd Quadrant and one in the 4th Quadrant.

The command $\tan^{-1}(1.5)$ gives $\theta = 56.31^\circ$.

The corresponding angles in the 2nd and 4th Quadrants are $180^\circ - 56.31^\circ = 123.69^\circ$ and $360^\circ - 56.31^\circ = 303.69^\circ$.

Thus, the solutions are 123.69° and 303.69° .

