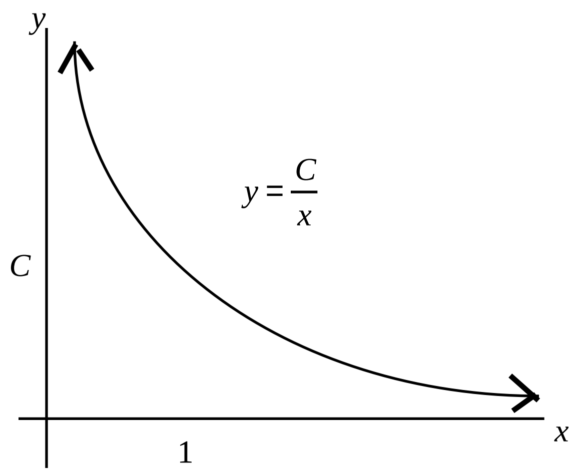
MATH 116 Inversely Proportional Functions

Let $y = \frac{C}{x}$ for $x > 0$. Then y is said to be *inversely proportional* to x. As x increases, then y decreases to 0. As x drops to 0, then y increases to $+\infty$.



To solve for the function, we need one measurement in order to find the constant C.

Example 1. Let y be inversely proportional to x where $y = 50$ when $x = 200$.

(a) Solve for y as a function of x. (b) What value of y occurs when $x = 40$?

(c) For which x is y at least 20?

Solution. (a) Let $y = \frac{C}{x}$. Then $50 = \frac{C}{200}$, and $C = 200 \times 50 = 10{,}000$. So we have $y = \frac{10{,}000}{x}$, for $x > 0$.

(b) Then when $x = 40$, we have $y = 10{,}000 / 40 = 250$.

(c) To make y at least 20, we solve the inequality $\frac{10{,}000}{x} \geq 20$ which gives $\frac{10{,}000}{20} \geq x$. Thus, $x \leq \frac{10{,}000}{20} = 500$. But due to the standard domain of inversely proportional functions, we must write $0 < x \leq 500$.

Example 2. Let y be inversely proportional to the *square of* x and suppose $y = 2$ when $x = 20$.

(a) Solve for y as a function of x. (b) For which x is y at most 32?

Solution. (a) Let $y = \frac{C}{x^2}$. Then $2 = \frac{C}{20^2}$ and $C = 2 \times 20^2 = 800$. Thus $y = \frac{800}{x^2}$ for $x > 0$.

(b) To make y at most 32, we solve the inequality $\frac{800}{x^2} \le 32$ which gives $\frac{800}{32} \le x^2$. Thus, $x^2 \ge \frac{800}{32} = 25$. Taking the square root, we have $x \ge 5$.

Example 3. At a fixed temperature, gas pressure P is inversely proportional to container volume V. Suppose that for a volume of 12 liters, the pressure is 4.5 psi .

(a) Find the pressure P as a function of volume V for this temperature.
(b) What happens to the pressure as volume decreases?
(c) What is the pressure for a volume of 15 liters?
(d) What volumes gives a pressure of at least 8 psi?

Solution. (a) Let $P = \frac{C}{V}$. Then $4.5 = \frac{C}{12}$, so $C = 4.5 \times 12 = 54$. Thus, $P = \frac{54}{V}$ for $V > 0$.

(b) As the volume decreases (to 0), the pressure goes up.

(c) For $V = 15$ liters, then $P = \frac{54}{15} = 3.6$ psi.

(d) Solve $\frac{54}{V} \ge 8$, which gives $\frac{54}{8} \ge V$. Thus we must have $0 < V \le 6.75$ liters.

Exercise

For objects moving in circles at a constant speed, the angular velocity ω is inversely proportional to the radius r. (The angular velocity measures how fast your angle is rotating as you move.) Suppose a radius of 10 miles gives an ω of 15° per hour.

(a) Find ω as a function of r.
(b) What is ω for a 5 miles radius? What is ω for a 20 mile radius?
(c) What happens to ω as r increases?
(d) What radii make ω more than 5° per hour?
(e) What radii make ω at most 50° per hour?

Solution

(a) Let $\omega = \dfrac{C}{r}$. Then $15 = \dfrac{C}{10}$ and $C = 150$. So $\omega = \dfrac{150}{r}$ for $r > 0$.

(b) For $r = 5$ miles, $\omega = \dfrac{150}{5} = 30^\circ$ per hour. For $r = 20$ miles, $\omega = \dfrac{150}{20} = 7.5^\circ$ per hour.

(c) ω decreases as r increases (Your angle moves more slowly as the radius increases provided the linear speed stays the same.)

(d) Solve $\dfrac{150}{r} > 5$ to obtain $30 > r$. So we must have $0 < r < 30$ miles.

(e) Solve $\dfrac{150}{r} \leq 50$ to obtain $3 \leq r$. So for $r \geq 3$ miles, we have $\omega \leq 50^\circ$ per hr.