Scaling Vector

Matrices

$$G[-2] = \left\{\left\{0, \frac{1}{80}\sqrt{\frac{7}{3}\left(23 - 6\sqrt{14}\right)}, -\frac{1}{80}\sqrt{3}\left(-1 + \sqrt{14}\right), -\frac{1}{80}\sqrt{\frac{151}{3} - 10\sqrt{14}}\right\},\right.$$
$$\left.\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\right\};$$

$$G[-1] = \left\{\left\{\frac{1}{40}\sqrt{\frac{151}{2} - 15\sqrt{14}}, -\frac{1}{80}\sqrt{\frac{71}{3} - 4\sqrt{14}}, \frac{1}{80}\left(-\sqrt{3} - 4\sqrt{42}\right), -\frac{1}{80}\sqrt{\frac{1351}{3} + 70\sqrt{14}}\right\},\right.$$
$$\left.\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\right\};$$

$$G[0] =$$
$$\left\{\left\{\frac{1}{\sqrt{2}}, -\frac{1}{80}\sqrt{\frac{71}{3} - 4\sqrt{14}}, \frac{1}{80}\left(\sqrt{3} + 4\sqrt{42}\right), -\frac{1}{80}\sqrt{\frac{1351}{3} + 70\sqrt{14}}\right\}, \left\{0, \frac{7}{8\sqrt{2}}, -\frac{\sqrt{\frac{7}{2}}}{8}, -\frac{1}{8}\right\},\right.$$
$$\left.\left\{0, \frac{3\sqrt{\frac{7}{2}}}{8}, \frac{1}{8\sqrt{2}}, 0\right\}, \left\{0, \frac{1}{80}\left(-16 - 3\sqrt{14}\right), -\frac{1}{40}\sqrt{\frac{905}{2} - 24\sqrt{14}}, -\frac{3\sqrt{7}}{40}\right\}\right\};$$

$$G[1] = \left\{\left\{\frac{1}{40}\sqrt{\frac{151}{2} - 15\sqrt{14}}, \frac{1}{80}\sqrt{\frac{7}{3}\left(23 - 6\sqrt{14}\right)}, \frac{1}{80}\sqrt{3}\left(-1 + \sqrt{14}\right), -\frac{1}{80}\sqrt{\frac{151}{3} - 10\sqrt{14}}\right\},\right.$$
$$\left\{\frac{\sqrt{\frac{3}{2}}}{4}, \frac{7}{8\sqrt{2}}, \frac{\sqrt{\frac{7}{2}}}{8}, -\frac{1}{8}\right\}, \left\{0, -\frac{3\sqrt{\frac{7}{2}}}{8}, \frac{1}{8\sqrt{2}}, 0\right\},$$
$$\left.\left\{\frac{3\sqrt{\frac{21}{2}}}{20}, \frac{1}{80}\left(-16 - 3\sqrt{14}\right), \frac{1}{40}\sqrt{\frac{905}{2} - 24\sqrt{14}}, -\frac{3\sqrt{7}}{40}\right\}\right\};$$

Function Values

```
Clear[phi];
```

```
phi[0] = {Sqrt[10], 0, 0, 0}
```

$$\left\{\sqrt{10}, 0, 0, 0\right\}$$

```
phi[x_] := If[x ≤ -1 || x ≥ 1, {0, 0, 0, 0}, phi[x]];
```

```
phi[3]
```

$$\{0, 0, 0, 0\}$$

```
fill[j_] := Do[phi[x] = Sqrt[2] Sum[G[i].phi[2 x - i], {i, -2, 1}], {x, -1, 1, 2^-j}];
```

```
fill[1]
```

```
fill[2]
```

```
fill[3]
```

```
fill[4]
```

```
fill[5]
```

```
fill[6]
```

```
fill[7]
```

```
pts[i_] := Table[{x, phi[x][[i]]}, {x, -1, 1, 2^-7}];
```

```
phi[-3/4] // FullSimplify
```

$$\left\{-\frac{1}{32}\sqrt{\frac{173}{2}-\frac{114\sqrt{14}}{5}},\ 0,\ 0,\ 0\right\}$$

```
phi[-1/2] // FullSimplify
```

$$\left\{\frac{1}{4}\sqrt{\frac{151}{10}-3\sqrt{14}},\ 0,\ 0,\ 0\right\}$$

```
phi[-1/4] // FullSimplify
```

$$\left\{-\frac{31+6\sqrt{14}}{32\sqrt{10}},\ 0,\ 0,\ 0\right\}$$

```
phi[0]
```

$$\left\{\sqrt{10},\ 0,\ 0,\ 0\right\}$$

```
phi[1/4] // FullSimplify
```

$$\left\{-\frac{31+6\sqrt{14}}{32\sqrt{10}},\ \frac{3\sqrt{\frac{15}{2}}}{8},\ \frac{3\sqrt{\frac{105}{2}}}{16},\ -\frac{1}{8}\sqrt{\frac{57}{2}+\frac{36\sqrt{14}}{5}}\right\}$$

```
phi[1/2] // FullSimplify
```

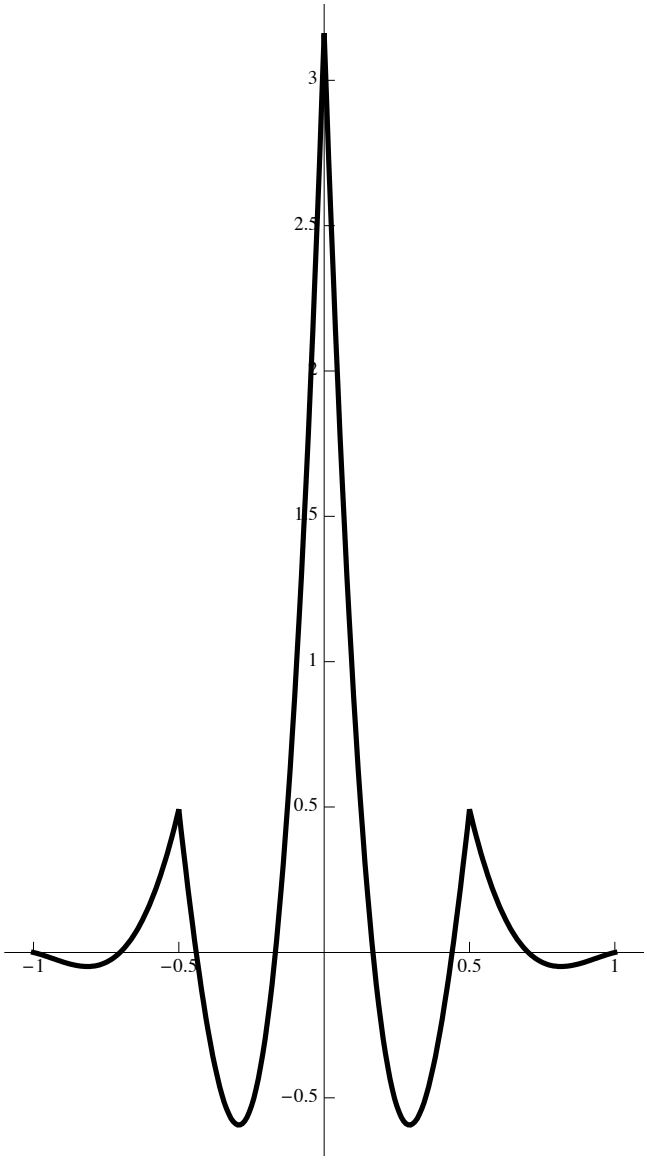
$$\left\{\frac{1}{4}\sqrt{\frac{151}{10}-3\sqrt{14}},\ \frac{\sqrt{\frac{15}{2}}}{2},\ 0,\ \frac{3\sqrt{\frac{21}{10}}}{2}\right\}$$

```
phi[3/4] // FullSimplify
```

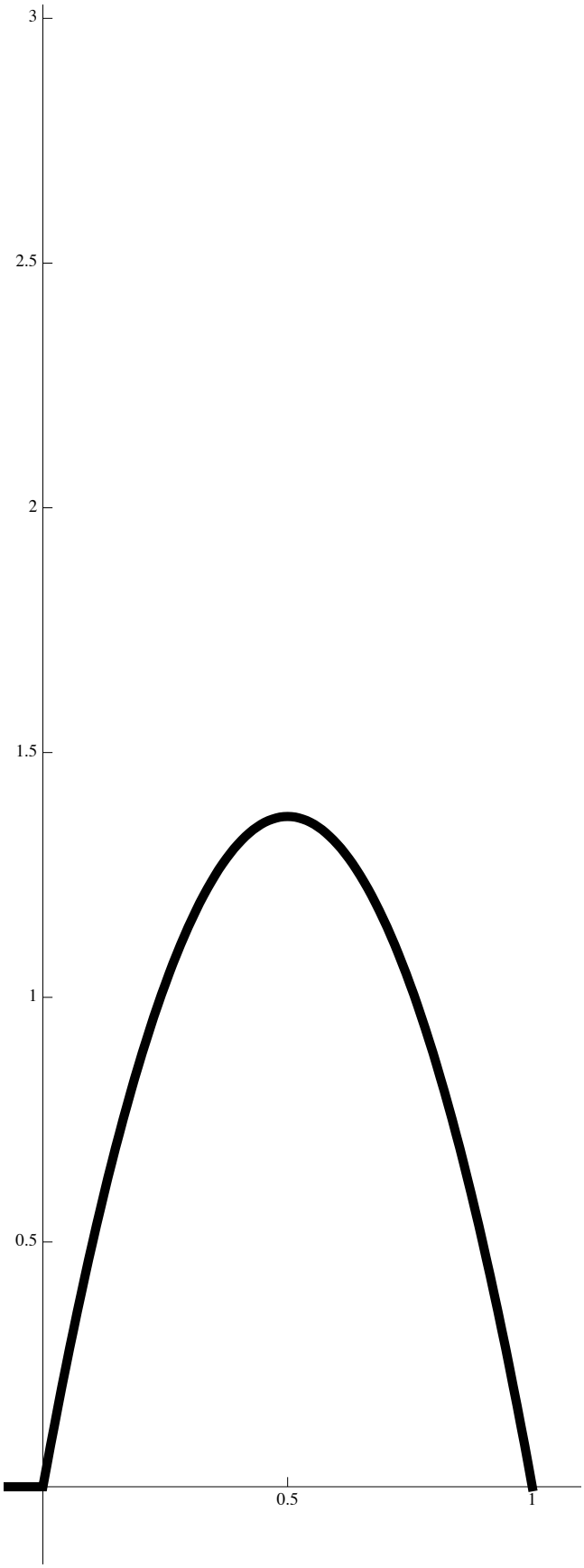
$$\left\{-\frac{1}{32}\sqrt{\frac{173}{2}-\frac{114\sqrt{14}}{5}},\ \frac{3\sqrt{\frac{15}{2}}}{8},\ -\frac{3\sqrt{\frac{105}{2}}}{16},\ -\frac{1}{8}\sqrt{\frac{57}{2}+\frac{36\sqrt{14}}{5}}\right\}$$

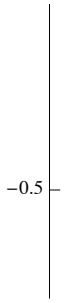
Graphs

```
Show[Graphics[{Black, Thickness[.008], Line[pts[1] // N]}],
 Axes → True, PlotRange → {{-1.1, 1.1}, {-0.7, √10 + .1}},
 Ticks → {{-1, -.5, .5, 1}, {-1.5, -1, -.5, .5, 1, 1.5, 2, 2.5, 3}}]
```

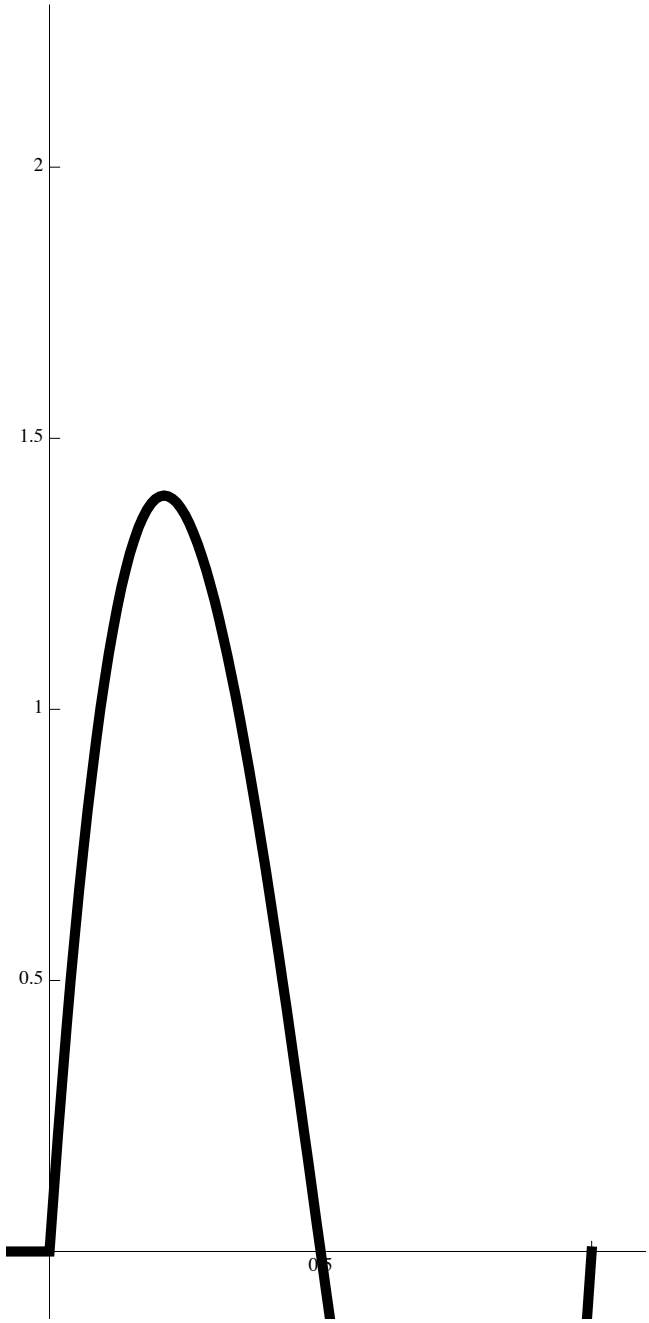


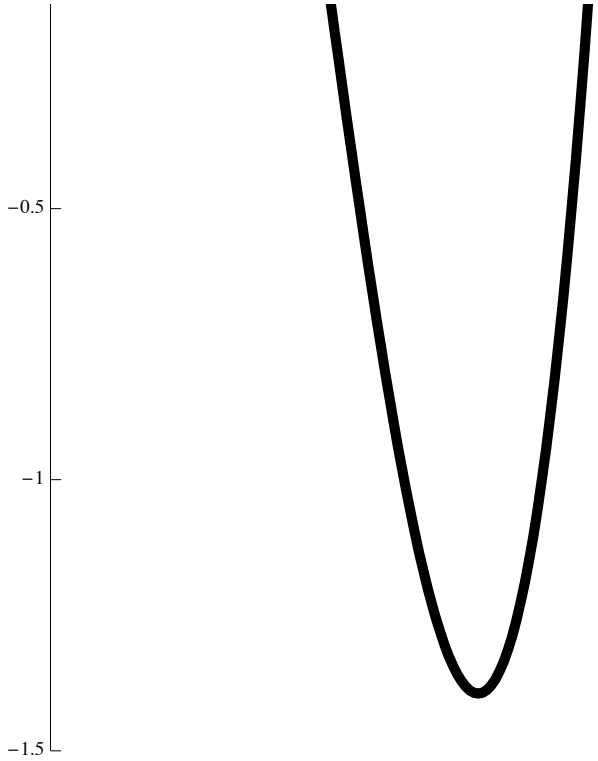

```
Show[Graphics[{Black, Thickness[.017], Line[pts[2] // N]}],
 Axes → True, PlotRange → {{0, 1.1}, {-0.7, √10 + .1}},
 Ticks → {{-1, -.5, .5, 1}, {-.5, .5, 1, 1.5, 2, 2.5, 3}}]
```


3
2.5
2
1.5
1
0.5
0.5
1

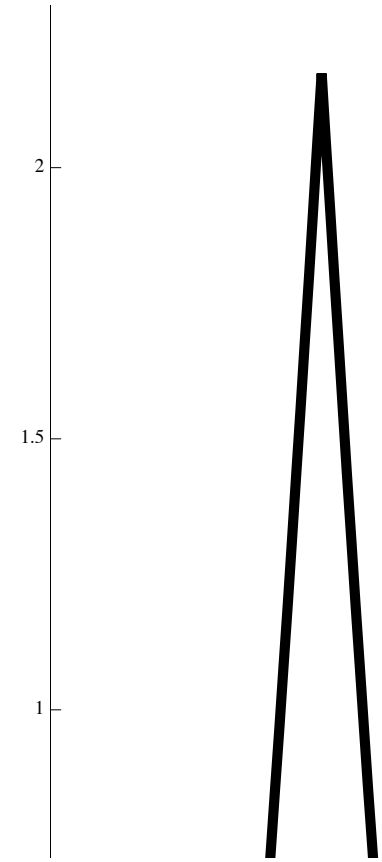


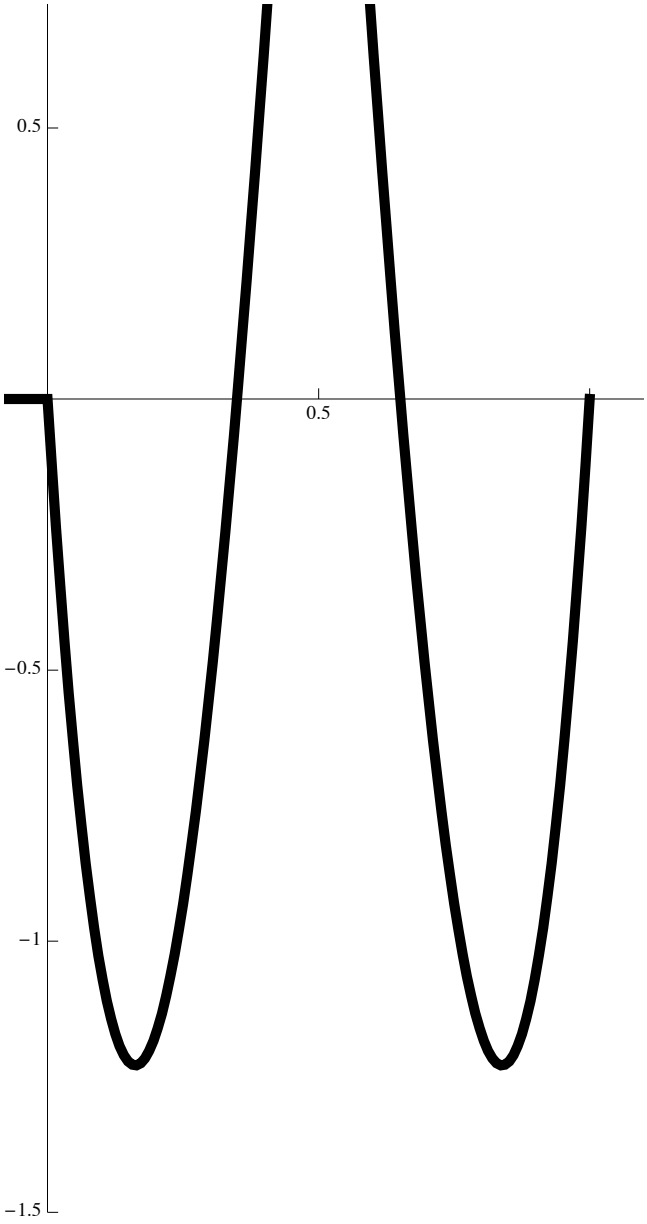

```
Show[Graphics[{Black, Thickness[.017], Line[pts[3] // N]}],
 Axes → True, PlotRange → {{0, 1.1}, {-1.5, 2.3}},
 Ticks → {{-1, -.5, .5, 1}, {-1.5, -1, -.5, .5, 1, 1.5, 2, 2.5, 3}}]
```






```
Show[Graphics[{Black, Thickness[.017], Line[pts[4] // N]}],
 Axes → True, PlotRange → {{0, 1.1}, {-1.5, 2.3}},
 Ticks → {{-1, -.5, .5, 1}, {-1.5, -1, -.5, .5, 1, 1.5, 2, 2.5, 3}}]
```





Multiwavelet

■ Matrices

$$H[-2] = \left\{\left\{0, -\frac{1}{80}\sqrt{\frac{7}{3}\left(23 - 6\sqrt{14}\right)}, \frac{1}{80}\sqrt{3}\left(-1 + \sqrt{14}\right), \frac{1}{80}\sqrt{\frac{151}{3} - 10\sqrt{14}}\right\},\right.$$

$$\left.\left\{0, -\frac{1}{40}\sqrt{\frac{161}{6} - 7\sqrt{14}}, \frac{1}{40}\sqrt{\frac{3}{2}}\left(-1 + \sqrt{14}\right), \frac{1}{40}\sqrt{\frac{151}{6} - 5\sqrt{14}}\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\right\};$$

$$H[-1] = \left\{\left\{-\frac{1}{40}\sqrt{\frac{151}{2}-15\sqrt{14}}\ ,\ \frac{1}{80}\sqrt{\frac{71}{3}-4\sqrt{14}}\ ,\ \frac{1}{80}\left(\sqrt{3}+4\sqrt{42}\right),\ \frac{1}{80}\sqrt{\frac{1351}{3}+70\sqrt{14}}\ \right\},\right.$$

$$\left\{\frac{1}{40}\left(5-3\sqrt{14}\right),\ \frac{1}{40}\sqrt{\frac{71}{6}-2\sqrt{14}}\ ,\ \frac{1}{80}\left(\sqrt{6}+8\sqrt{21}\right),\ \frac{1}{40}\sqrt{\frac{1351}{6}+35\sqrt{14}}\ \right\},$$

$$\left.\{0,0,0,0\},\{0,0,0,0\}\right\};$$

$$H[0] = \left\{\left\{\frac{1}{\sqrt{2}},\ \frac{1}{80}\sqrt{\frac{71}{3}-4\sqrt{14}}\ ,\ \frac{1}{80}\left(-\sqrt{3}-4\sqrt{42}\right),\ \frac{1}{80}\sqrt{\frac{1351}{3}+70\sqrt{14}}\ \right\},\right.$$

$$\left\{0,-\frac{1}{40}\sqrt{\frac{71}{6}-2\sqrt{14}}\ ,\ \frac{1}{80}\left(\sqrt{6}+8\sqrt{21}\right),\ -\frac{1}{40}\sqrt{\frac{1351}{6}+35\sqrt{14}}\ \right\},$$

$$\left.\left\{0,\frac{1}{40}\left(3-\sqrt{14}\right),\ \frac{1}{40}\sqrt{65+6\sqrt{14}}\ ,\ \frac{1}{20}\sqrt{\frac{239}{2}-15\sqrt{14}}\ \right\},\left\{0,-\frac{1}{8\sqrt{6}},\ \frac{\sqrt{\frac{21}{2}}}{8},\ \frac{1}{\sqrt{3}}\right\}\right\};$$

$$H[1] = \left\{\left\{-\frac{1}{40}\sqrt{\frac{151}{2}-15\sqrt{14}}\ ,\ -\frac{1}{80}\sqrt{\frac{7}{3}\left(23-6\sqrt{14}\right)}\ ,\ -\frac{1}{80}\sqrt{3}\ \left(-1+\sqrt{14}\right),\ \frac{1}{80}\sqrt{\frac{151}{3}-10\sqrt{14}}\ \right\},\right.$$

$$\left\{\frac{1}{40}\left(-5+3\sqrt{14}\right),\ \frac{1}{40}\sqrt{\frac{161}{6}-7\sqrt{14}}\ ,\ \frac{1}{40}\sqrt{\frac{3}{2}}\ \left(-1+\sqrt{14}\right),\ -\frac{1}{40}\sqrt{\frac{151}{6}-5\sqrt{14}}\ \right\},$$

$$\left\{\frac{1}{20}\sqrt{3}\ \left(5+\sqrt{14}\right),\ \frac{1}{40}\left(3-\sqrt{14}\right),\ \frac{1}{40}\left(-\sqrt{2}-3\sqrt{7}\right),\ \frac{1}{20}\sqrt{\frac{239}{2}-15\sqrt{14}}\ \right\},$$

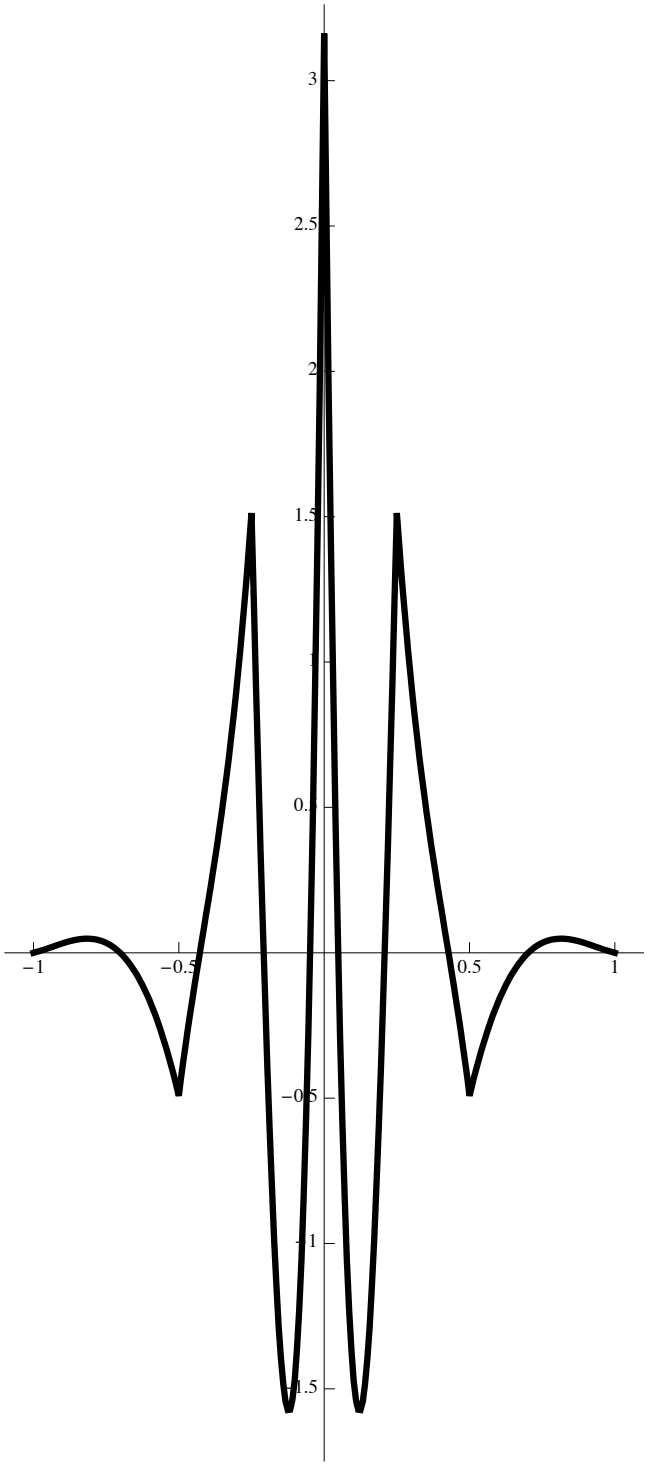
$$\left.\left\{0,\frac{1}{8\sqrt{6}},\ \frac{\sqrt{\frac{21}{2}}}{8},\ -\frac{1}{\sqrt{3}}\right\}\right\};$$

■ Graphs

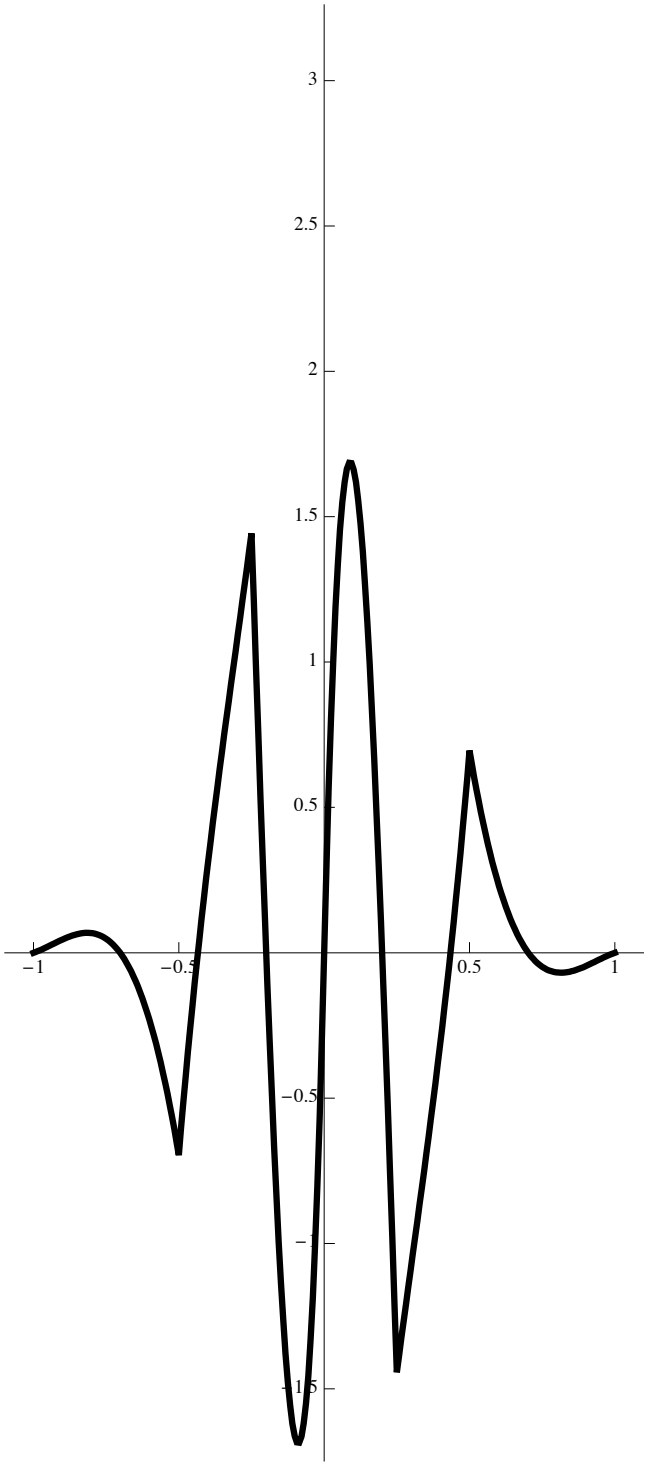
```
Do[psi[x] = Sqrt[2] Sum[H[i].phi[2 x - i], {i, -2, 1}], {x, -1, 1, 2^-7}];

pts[i_] := Table[{x, psi[x][[i]]} // N, {x, -1, 1, 2^-7}];
```

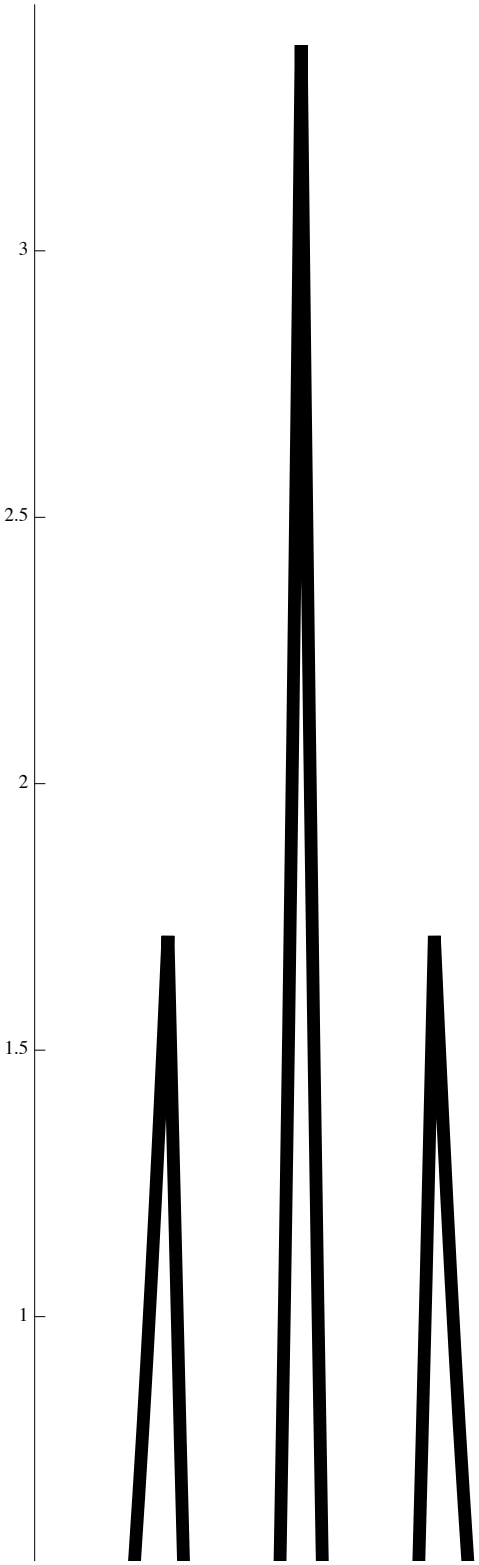
```
Show[Graphics[{Black, Thickness[.01], Line[pts[1]]}],
 Axes → True, PlotRange → {{-1.1, 1.1}, {-1.75, √10 + .1}},
 Ticks → {{-1, -.5, .5, 1}, {-1.5, -1, -.5, .5, 1, 1.5, 2, 2.5, 3}}]
```

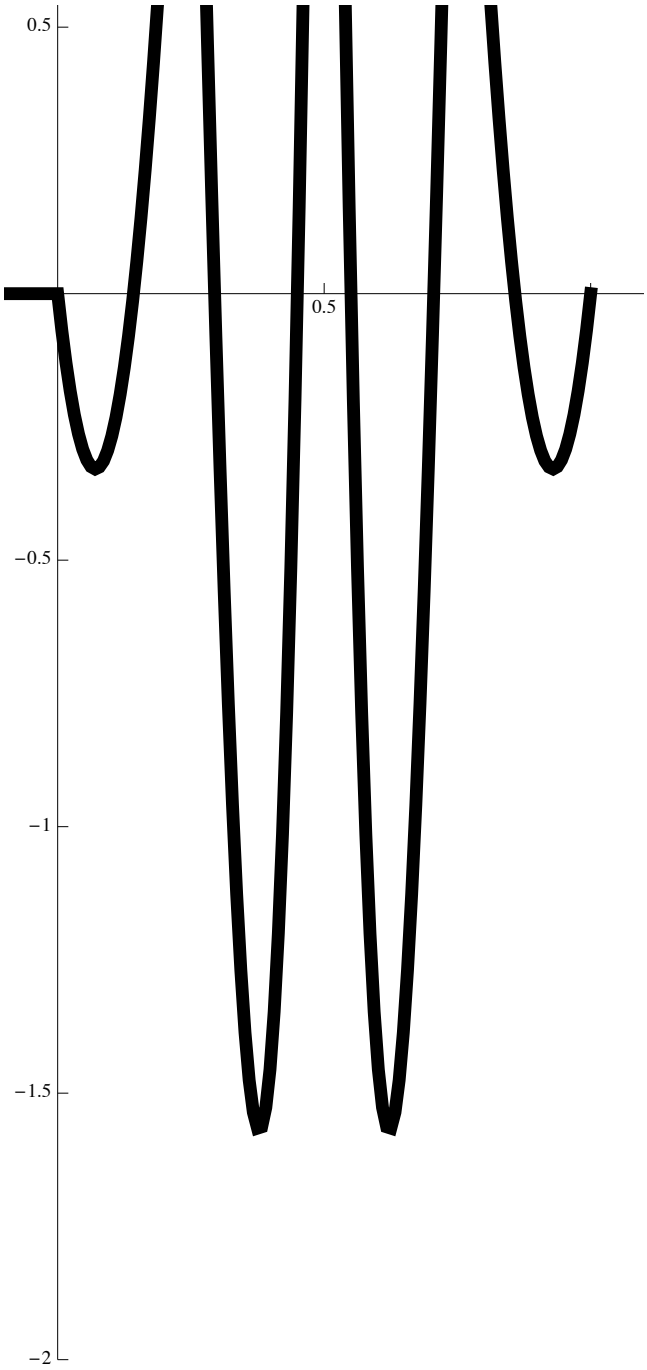


```
Show[Graphics[{Black, Thickness[.01], Line[pts[2]]}],
 Axes → True, PlotRange → {{-1.1, 1.1}, {-1.75, √10 + .1}},
 Ticks → {{-1, -.5, .5, 1}, {-1.5, -1, -.5, .5, 1, 1.5, 2, 2.5, 3}}]
```

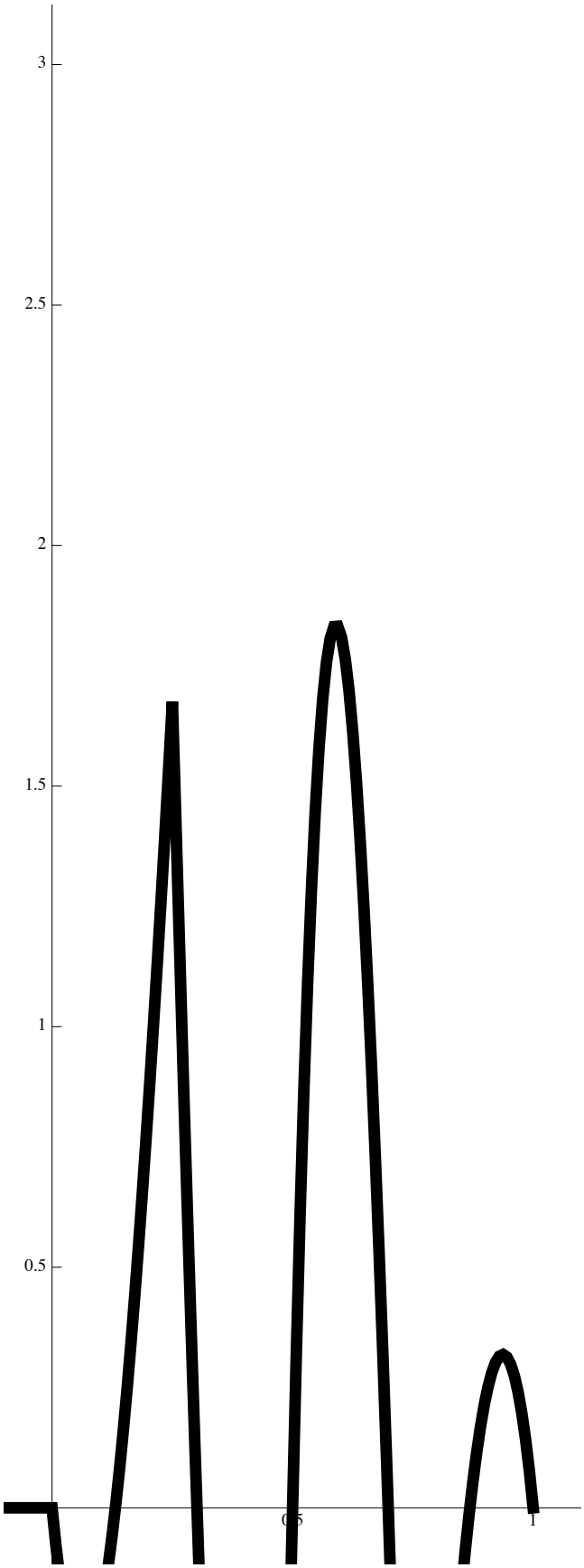


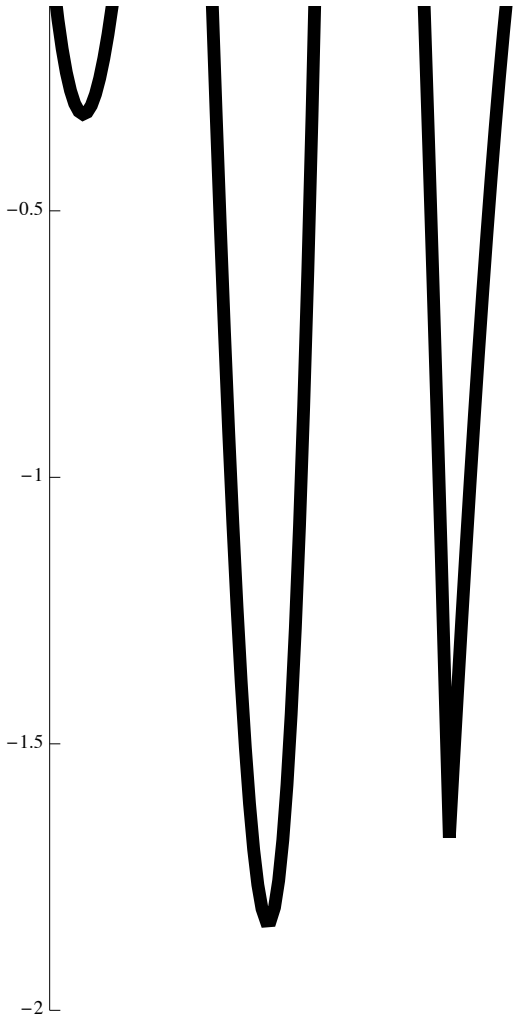
```
Show[Graphics[{Black, Thickness[.02], Line[pts[3]]}],
 Axes → True, PlotRange → {{-.1, 1.1}, {-2, √10 + .3}},
 Ticks → {{-1, -.5, .5, 1}, {-2, -1.5, -1, -.5, .5, 1, 1.5, 2, 2.5, 3}}]
```






```
Show[Graphics[{Black, Thickness[.02], Line[pts[4]]}],
 Axes → True, PlotRange → {{-.1, 1.1}, {-2, √10 + .3}},
 Ticks → {{-1, -.5, .5, 1}, {-2, -1.5, -1, -.5, .5, 1, 1.5, 2, 2.5, 3}}]
```


3
2.5
2
1.5
1
0.5
0.5
1


−0.5
−1
−1.5
−2