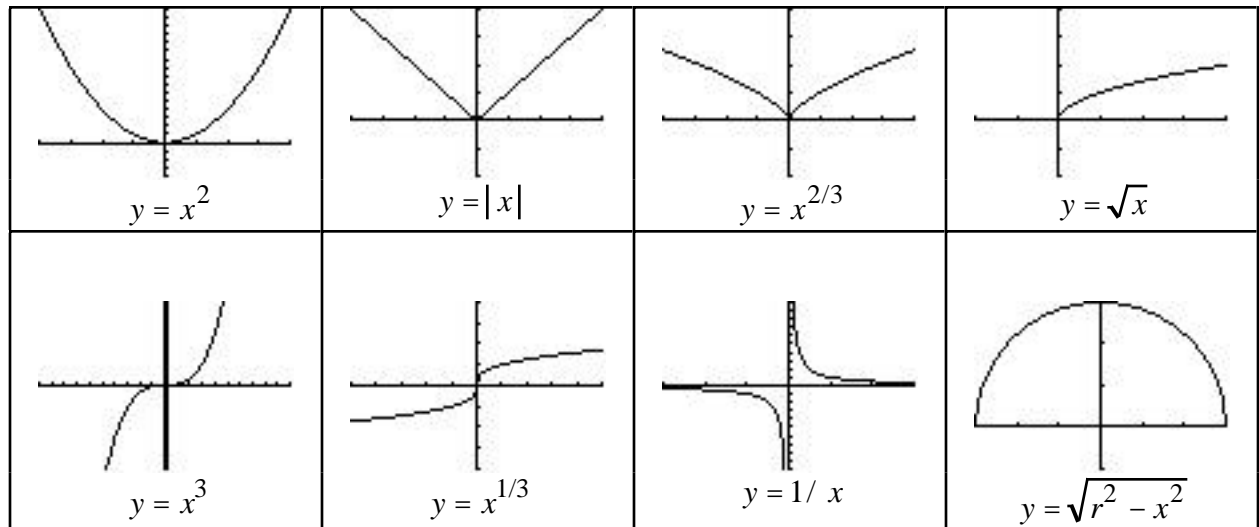


MATH 116

Translations of $y = f(x)$ 

- I. Multiplying by a positive constant c steepens the graph (for $c > 1$) or widens the graph (for $0 < c < 1$) II. Multiplying by -1 flips the graph upside down.
 III. $y = f(x - h) + k$ translates the graph to a new origin of (h, k) .

- (i) Find the x and y -intercepts. (Show all algebra in finding the x -intercepts.)
 (ii) Graph and label the translation point of the origin. (iii) State the domain and range.
 (iv) State where the function is increasing and where it is decreasing.

(a) $f(x) = -4(x + 6)^2 + 9$	(b) $f(x) = \frac{1}{3}(x - 8)^2 - 12$
(c) $f(x) = \frac{1}{2} x + 10 - 8$	(d) $f(x) = -3 x - 6 + 15$
(e) $f(x) = 9(x + 1)^{2/3} - 16$	(f) $f(x) = -\frac{1}{12}(x - 8)^{2/3} + 3$
(g) $f(x) = -\frac{1}{4}(x - 4)^3 - 2$	(h) $f(x) = 2(x + 27)^3 + 54$
(i) $f(x) = -4(x - 8)^{1/3} - 12$	(j) $f(x) = \frac{1}{3}(x + 64)^{1/3} + 3$
(k) $f(x) = 6\sqrt{x + 9} - 30$	(l) $f(x) = -\frac{1}{4}\sqrt{x - 10} + 5$
(m) $f(x) = -\frac{6}{x + 4} + 12$	(n) $f(x) = \frac{10}{x - 2} - 8$
(o) $y = -\sqrt{36 - (x - 9)^2} + 8$	(p) $y = \sqrt{100 - (x + 5)^2} - 6$

Solutions

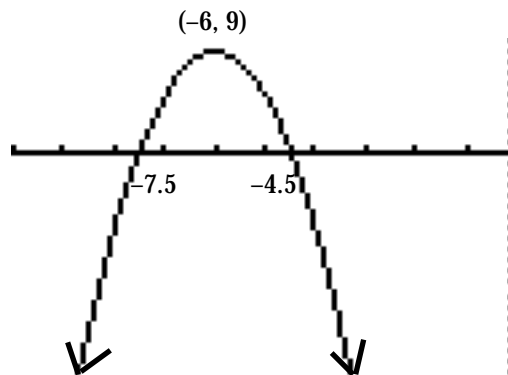
(a) $f(x) = -4(x+6)^2 + 9$ Shift to $(-6, 9)$.

y intercept: $(0, -135)$

For the x intercepts: $-4(x+6)^2 + 9 = 0$

$$-4(x+6)^2 = -9 \quad (x+6)^2 = \frac{9}{4} \quad x+6 = \pm \frac{3}{2}$$

$$x = -6 \pm \frac{3}{2} \quad x = -7.5 \text{ and } x = -4.5$$

Domain: $(-\infty, \infty)$ Range: $(-\infty, 9]$  f increases for x in $(-\infty, -6)$ and decreases for x in $(-6, \infty)$.

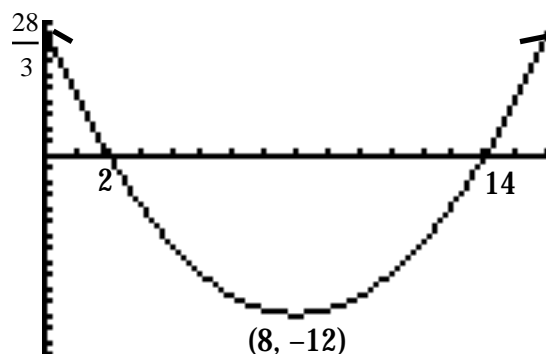
(b) $f(x) = \frac{1}{3}(x-8)^2 - 12$ Shift to $(8, -12)$.

y intercept: $(0, 28/3)$

For the x intercepts: $\frac{1}{3}(x-8)^2 - 12 = 0$

$$(x-8)^2 = 36 \quad x-8 = \pm 6 \quad x = 8 \pm 6$$

$$x = 2 \text{ and } x = 14$$

Domain: $(-\infty, \infty)$ Range: $[-12, \infty)$  f decreases for x in $(-\infty, 8)$ and increases for x in $(8, \infty)$.

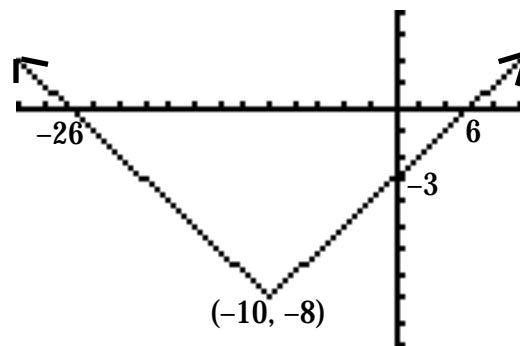
(c) $f(x) = \frac{1}{2}|x+10| - 8$ Shift to $(-10, -8)$.

y intercept: $(0, -3)$

For the x intercepts: $\frac{1}{2}|x+10| - 8 = 0$

$$|x+10| = 16 \quad x+10 = \pm 16 \quad x = -10 \pm 16$$

$$x = -26 \text{ and } x = 6$$

Domain: $(-\infty, \infty)$ Range: $[-8, \infty)$  f decreases for x in $(-\infty, -10)$ and increases for x in $(-10, \infty)$.

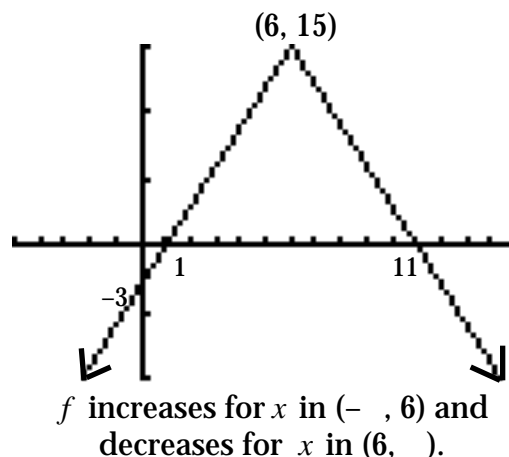
(d) $f(x) = -3|x - 6| + 15$ Shift to (6, 15).

y intercept: (0, -3)

For the x intercepts: $-3|x - 6| + 15 = 0$
 $-3|x - 6| = -15$ $|x - 6| = 5$ $x - 6 = \pm 5$
 $x = 6 \pm 5$

$x = 1$ and $x = 11$

Domain: $(-\infty, \infty)$ Range: $(-\infty, 15]$



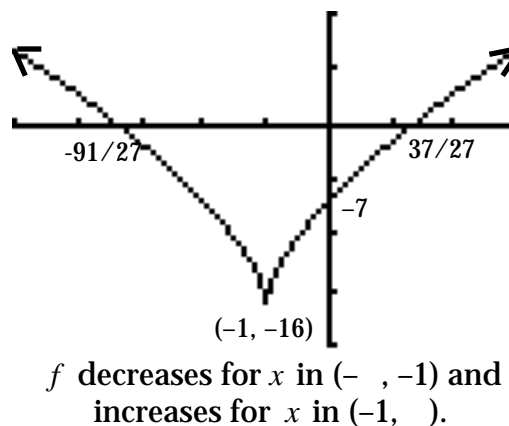
(e) $f(x) = 9(x + 1)^{2/3} - 16$ Shift to (-1, -16).

y intercept: (0, -7)

For the x intercepts: $9(x + 1)^{2/3} - 16 = 0$
 $(x + 1)^{2/3} = 16/9$ $(x + 1)^{1/3} = \pm 4/3$
 $x + 1 = \pm 64/27$ $x = -1 \pm 64/27$

$x = -91/27$ and $x = 37/27$

Domain: $(-\infty, \infty)$ Range: $[-16, \infty)$



(f) $f(x) = -\frac{1}{12}(x - 8)^{2/3} + 3$ Shift to (8, 3).

y intercept: (0, 8/3)

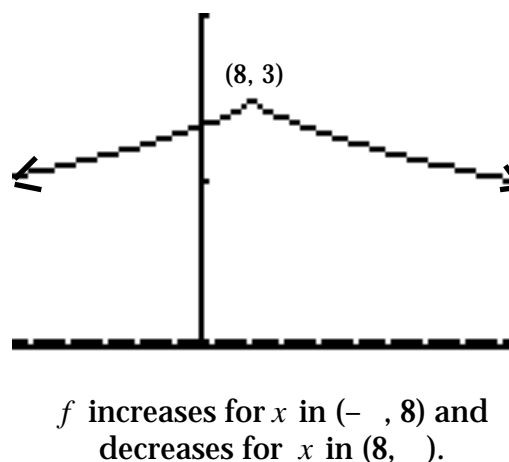
For the x intercepts: $-\frac{1}{12}(x - 8)^{2/3} + 3 = 0$

$-\frac{1}{12}(x - 8)^{2/3} = -3$ $(x - 8)^{2/3} = 36$

$(x - 8)^{1/3} = \pm 6$ $x - 8 = \pm 216$ $x = 8 \pm 216$

$x = -208$ and $x = 224$

Domain: $(-\infty, \infty)$ Range: $(-\infty, 3]$



(g) $f(x) = -\frac{1}{4}(x-4)^3 - 2$ Shift to (4, -2).

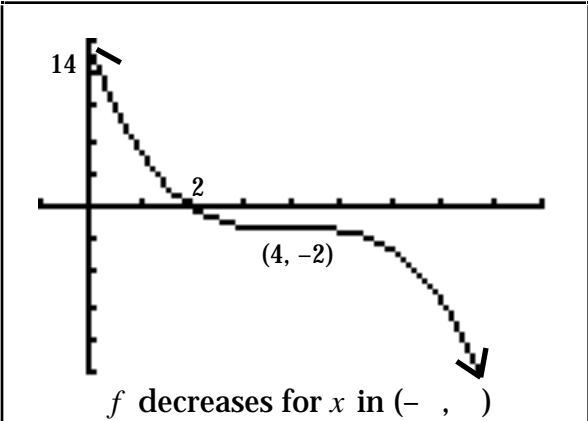
y intercept: (0, 14)

For the x intercept: $-\frac{1}{4}(x-4)^3 - 2 = 0$

$$-\frac{1}{4}(x-4)^3 = 2 \quad (x-4)^3 = -8 \quad x-4 = -2$$

$$x = 2$$

Domain: (- ,) Range: (- ,)



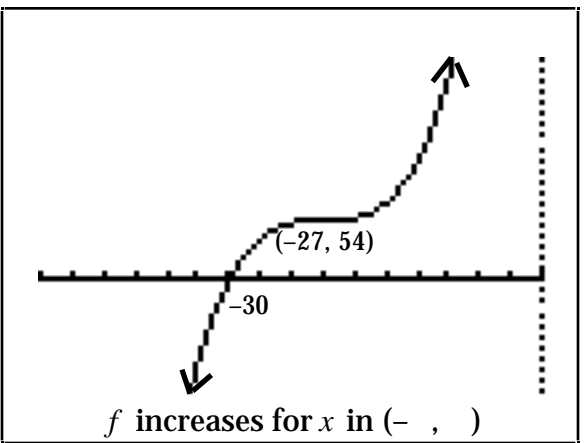
(h) $f(x) = 2(x+27)^3 + 54$ Shift to (-27, 54).

y intercept: (0, 39420)

For the x intercept: $2(x+27)^3 + 54 = 0$

$$(x+27)^3 = -27 \quad x+27 = -3 \quad x = -30$$

Domain: (- ,) Range: (- ,)



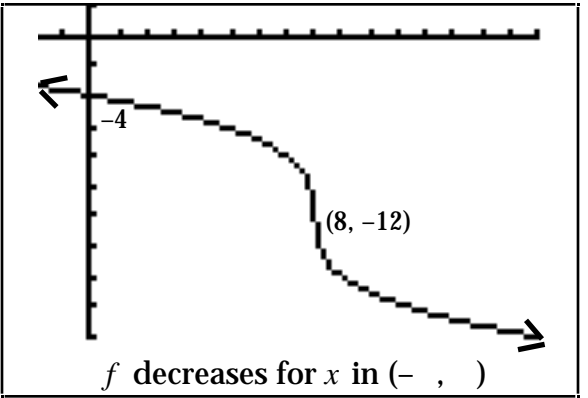
(i) $f(x) = -4(x-8)^{1/3} - 12$ Shift to (8, -12).

y intercept: (0, -4)

For the x intercept: $-4(x-8)^{1/3} - 12 = 0$

$$(x-8)^{1/3} = -3 \quad x-8 = -27 \quad x = -19$$

Domain: (- ,) Range: (- ,)



(j) $f(x) = \frac{1}{3}(x+64)^{1/3} + 3$ Shift to (-64, 3).

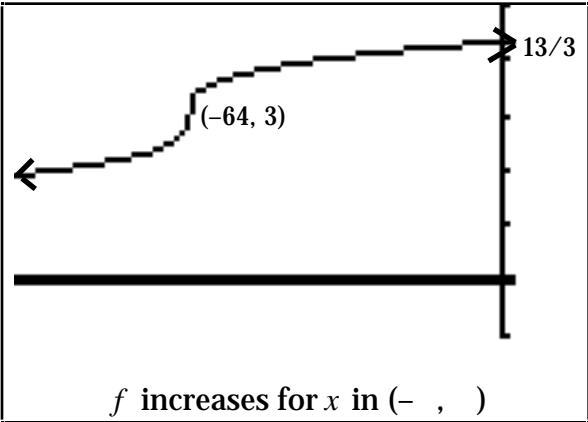
y intercept: (0, 13/3)

For the x intercept: $\frac{1}{3}(x+64)^{1/3} + 3 = 0$

$$(x+64)^{1/3} = -9 \quad x+64 = -729$$

$$x = -793$$

Domain: (- ,) Range: (- ,)

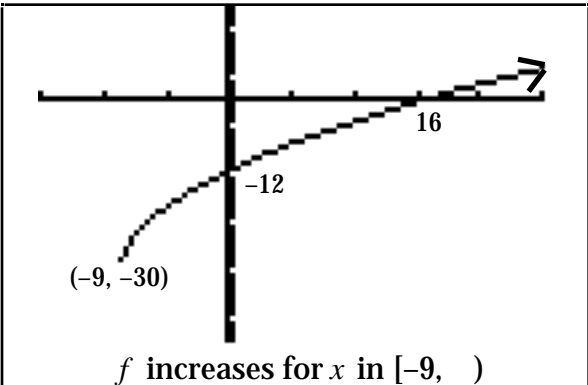


(k) $f(x) = 6\sqrt{x+9} - 30$ Shift to $(-9, -30)$.

y intercept: $(0, -12)$

x intercept: $6\sqrt{x+9} - 30 = 0 \quad 6\sqrt{x+9} = 30$
 $\sqrt{x+9} = 5 \quad x+9 = 25 \quad x = 16$

Domain: $[-9, \infty)$ Range: $[-30, \infty)$

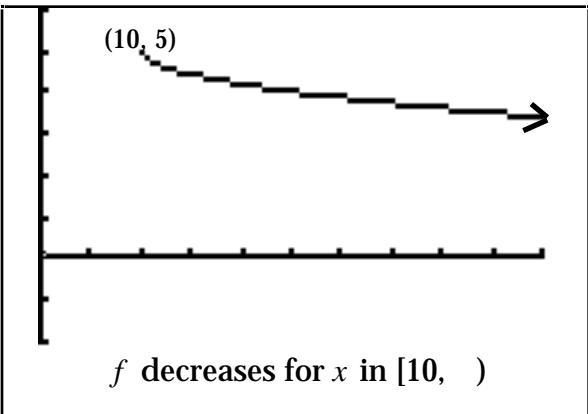


(l) $f(x) = -\frac{1}{4}\sqrt{x-10} + 5$ Shift to $(10, 5)$.

y intercept: None

x intercept: $-\frac{1}{4}\sqrt{x-10} + 5 = 0$
 $-\frac{1}{4}\sqrt{x-10} = -5 \quad \sqrt{x-10} = 20$
 $x-10 = 400 \quad x = 410$

Domain: $[10, \infty)$ Range: $(-\infty, 5]$

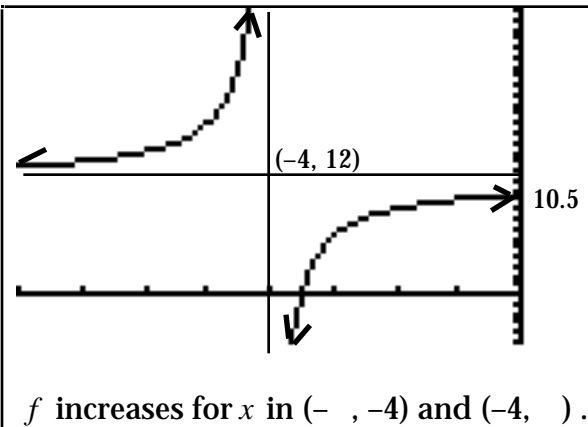


(m) $f(x) = -\frac{6}{x+4} + 12$ Shift to $(-4, 12)$.

y intercept: $(0, 10.5)$

For the x intercept: $-\frac{6}{x+4} + 12 = 0$
 $12 = \frac{6}{x+4} \quad x+4 = \frac{6}{12} \quad x = \frac{6}{12} - 4 = -3.5$

Domain: $x \neq -4$ Range: $y \neq 12$

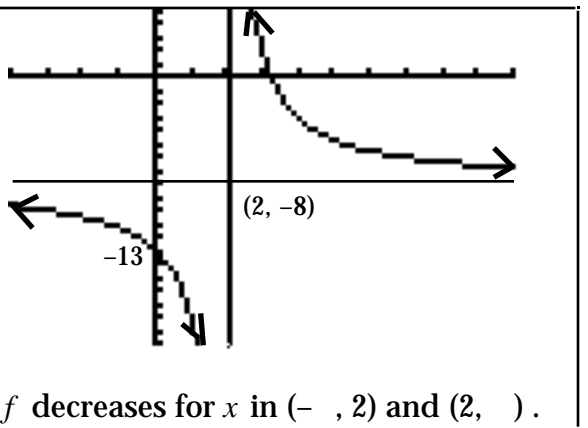


(n) $f(x) = \frac{10}{x-2} - 8$ Shift to $(2, -8)$.

y intercept: $(0, -13)$

For the x intercept: $\frac{10}{x-2} - 8 = 0$
 $\frac{10}{x-2} = 8 \quad \frac{10}{8} = x-2 \quad x = 2 + \frac{10}{8} = 3.25$

Domain: $x \neq 2$ Range: $y \neq -8$



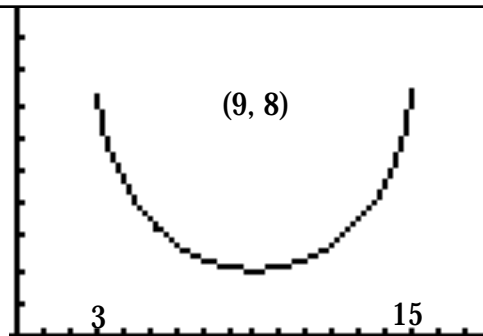
$$(o) \quad y = -\sqrt{36 - (x - 9)^2} + 8$$

Lower semicircle of radius 6 with center at (9, 8).

y intercept: None

x intercepts: None

Domain: [3, 15] Range: [2, 8]



f decreases for x in [3, 9) and increases for x in (9, 15].

$$(p) \quad y = \sqrt{100 - (x + 5)^2} - 6$$

Upper semicircle of radius 10 with center at (-5, -6).

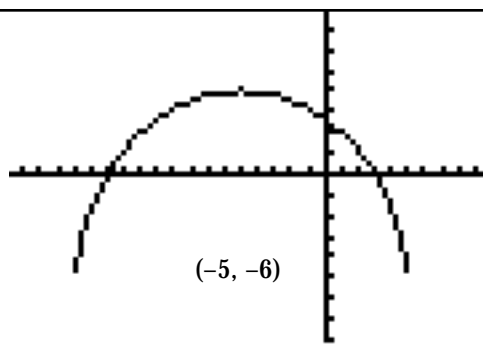
y intercept: (0, $\sqrt{75} - 6$)

For x intercepts: $\sqrt{100 - (x + 5)^2} - 6 = 0$

$$100 - (x + 5)^2 = 36 \quad 64 = (x + 5)^2$$

$$x + 5 = \pm 8 \quad x = -5 \pm 8 \quad x = -13 \text{ or } 3$$

Domain: [-15, 5] Range: [-6, 4]



f increases for x in [-15, -5) and decreases for x in (-5, 5].