MATH 116 Practice Test 2

1. Let $y = 4x^2 - 24x + 6$. (a) Find the vertex. (b) Give the y-intercept.

(c) Use the *quadratic formula* to find the x-intercepts. Then give the decimal form.

(d) Graph y. Label the intercepts and vertex.

(e) Solve the equations $y = -14$, $y = -5$, and $y = 34$ by factoring.

(f) For which x is $y \le 34$? When is $y > 34$? *Show* and *explain* the results on a graph.

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2. (i) Angular velocity ω is inversely proportional to the radius r. Suppose that for a radius of 10 ft, the angular velocity is 45 degrees per sec.

(a) Find the angular velocity as a function of radius. Graph the function with each axis labeled by name, symbol, and unit and with a specific point plotted. State the domain and range in *inequality* form.
(b) What is the angular velocity for a radius of 15 ft?
(c) Solve for the *radii* that give an angular velocity of *at most* 60 degrees per sec.
(d) What happens to the angular velocity as radius increases? As radius decreases?

(ii) Density ρ is inversely proportional to volume V. Suppose for a volume of 30 cm^3, the density is 1800 ppm.

(a) Find the density as a function of volume. Graph the function with each axis labeled by name, symbol, and unit and with a specific point plotted. State the domain and range in *inequality* form.
(b) What is the density for a volume of 10 cm^3?
(c) Solve for the *volumes* that give a density of *at least* 900 ppm.
(d) What happens to the density as volume decreases? As volume increases?

Henceforth, give proper algebraic solutions. Specify units throughout and show all work in your conversions.

Falling Objects: **Height**: $h(t) = -16t^2 + v_0 t + h_0$ ft **Velocity:** $v(t) = -32t + v_0$ ft/s

3. (i) An (aerodynamic) object is dropped from 22 feet high.

(a) Give the height function in this case. (b) *Solve* for the time T it takes to fall 12 ft.
(c) Give the speed of the object in mph at the time found in Part (b).

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(ii) An (aerodynamic) object is dropped and it takes 2.24 seconds to hit the ground.

(a) Use the appropriate height function to solve for the original height of the object.
(b) Find the impact speed in mph.

4. An object is thrown straight upward from 96 ft at 80 ft / sec.

(a) With no air resistance, what are the object's height and velocity functions?
(b) Solve for the maximum height of the object.
(c) Solve for the times when the object's speed equals 32 ft / sec.
(d) Give the *total time* it takes for the object hit the ground.
(e) Compute the object's impact speed upon its hitting the ground.

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5. A ball bounces straight upward to a maximum height of 9 feet.

(a) What are the height and velocity functions in this case?
(b) With what initial velocity did it spring from the floor? What is the time T that it takes to reach its maximum height? (Give a complete algebraic solution.)
(c) With the same initial velocity found in Part (a), how high would the ball bounce on Rigel where gravity is about 20 ft / s^2, and how long would it take to get there?

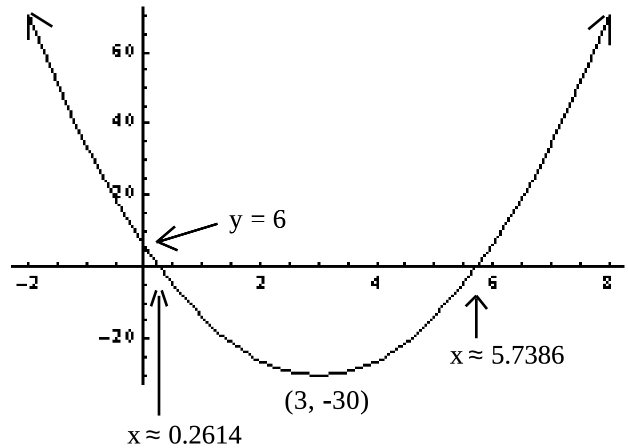
Solutions

1. Let $y = 4x^2 - 24x + 6$ (a) The x-coordinate of the vertex is $x = \frac{-b}{2a} = \frac{24}{8} = 3$. Then when $x = 3$, $y = -30$. So the vertex is (3, –30) (b) The y-int. is $y = 6$.

(c) $x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(4)(6)}}{2(4)}$

$= \frac{24 \pm \sqrt{480}}{8} = 3 \pm \frac{4\sqrt{30}}{8} = 3 \pm \frac{\sqrt{30}}{2}$

(or $x \approx 0.2614$ and $x \approx 5.7386$)

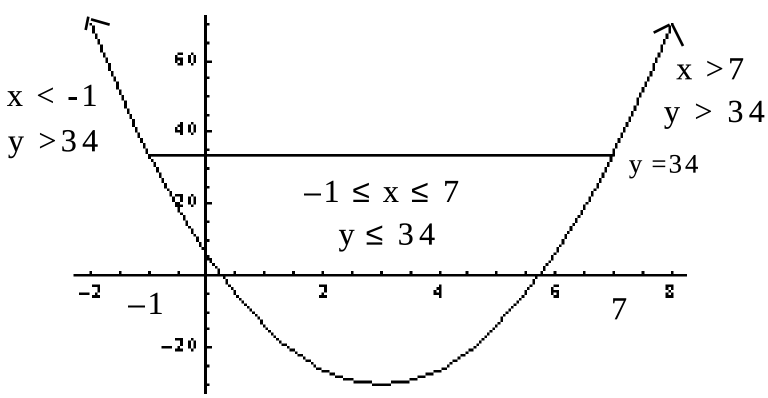


(e) $4x^2 - 24x + 6 = -14 \rightarrow 4x^2 - 24x + 20 = 0 \rightarrow x^2 - 6x + 5 = 0 \rightarrow (x-5)(x-1) = 0$
$\rightarrow x = 5$ or $x = 1$

$4x^2 - 24x + 6 = -5 \rightarrow 4x^2 - 24x + 11 = 0 \rightarrow (2x-1)(2x-11) = 0 \rightarrow x = \frac{1}{2}$ or $x = \frac{11}{2}$

$4x^2 - 24x + 6 = 34 \rightarrow 4x^2 - 24x - 28 = 0 \rightarrow x^2 - 6x - 7 = 0 \rightarrow (x+1)(x-7) = 0$
$\rightarrow x = -1$ or $x = 7$.

(f) $y \le 34$ for $-1 \le x \le 7$, and $y > 34$ for x in (–∞, –1) (7, ∞).



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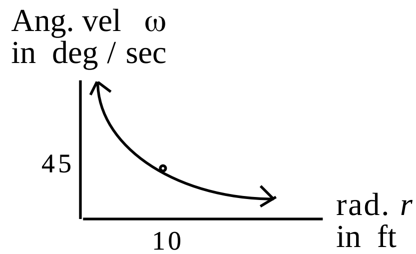
2. (i) First, $\omega = \dfrac{C}{r}$. Then $45 = \dfrac{C}{10}$; so that $C = 450$. Thus,

$$\omega = \frac{450}{r} \text{ for } r > 0.$$



Domain: $r > 0$ Range: $\omega > 0$

(b) For $r = 15$ ft, then $\omega = \dfrac{450}{15} = 30$ deg/sec.

(c) For ω to be at most 60 deg per sec, we solve $\dfrac{450}{r} \le 60$ which gives $\dfrac{450}{60} \le r$. So we need $r \ge 7.5$ ft.

(d) As the radius r increases, then ω decreases. As r decreases, then ω increases.

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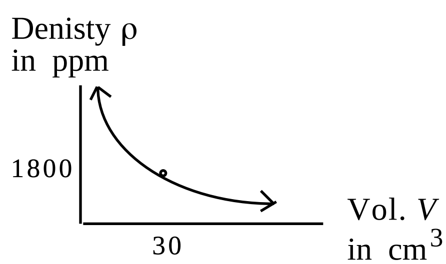
(ii) (a) First, $\rho = \dfrac{C}{V}$. Then $1800 = \dfrac{C}{30}$ so that $C = 1800 \times 30 = 54,000$.

Thus, $\rho = \dfrac{54,000}{V}$ for $V > 0$.



Domain: $V > 0$ Range: $\rho > 0$

(b) For $V = 10$ cm^3, then $\rho = \dfrac{54,000}{10} = 5400$ ppm.

(c) For ρ to be at least 900 ppm, we solve $\dfrac{54,000}{V} \ge 900$ which gives $\dfrac{54,000}{900} \ge V$. So we need $0 < V \le 60$ cm^3. (Note: V must also be positive.)

(d) As volume decreases, then density increases. As volume increases, then density decreases.

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3. (i) (a) Here, $h(t) = -16t^2 + 22$ and $v(t) = -32t$. The *speed* function is $s(t) = 32t$.
(b) Upon falling 12 feet at time T, the height becomes 22 – 12 = 10 ft. Thus, we solve $h(T) = 10 \to -16T^2 + 22 = 10 \to -16T^2 = -12 \to T^2 = \dfrac{12}{16} \to T = \dfrac{\sqrt{12}}{4} \approx$ **0.866 sec**

(b) The speed at this time is $32 \times \dfrac{\sqrt{12}}{4} = 8\sqrt{12}$ ft/s $\to \times 3600/5280 \approx$ **18.895 mph**.

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(ii) (a) Using $v_0 = 0$, we have $h(t) = -16t^2 + h_0$ and $v(t) = -32t$. Then, $h(2.24) = 0$; so $-16(2.24)^2 + h_0 = 0 \rightarrow h_0 = 16(2.24)^2 \approx$ **80.28 ft**.

(b) The impact velocity is –32(2.24) = –71.68 ft / s; so the impact *speed* is about 71.68 ft / s × 3600 / 5280 ≈ **48.87 mph**.

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4. (a) The height is $h(t) = -16t^2 + 80t + 96$ and the velocity is $v(t) = -32t + 80$

(b) At the maximum height, the velocity becomes 0. So we solve $v(t) = 0 \rightarrow -32t + 80 = 0 \rightarrow t = 2.5$ sec, which gives the time of reaching the maximum height. Then the max ht is $h(2.5) = -16(2.5)^2 + 80(2.5) + 96 =$ **196 ft**.

(c) For the speed to be 32 ft / s, the velocity is ±32 ft / s. So solve $v(t) = \pm 32 \rightarrow -32t + 80 = \pm 32 \rightarrow -32t = -48$ or $-32t = -112$. Thus, at times $t =$ **1.5 sec** and $t =$ **3.5 sec** the speed is 32 ft / s.

(d) The maximum height is 196 feet, which took 2.5 seconds to reach. To fall 196 feet back to the ground, it takes an *additional* $\frac{\sqrt{196}}{4} = 3.5$ seconds. The *total* time to hit the ground is then $t = 2.5 + 3.5 =$ **6 sec**.

(e) The impact speed upon falling 196 ft from the max ht is $32 \times 3.5 =$ **112 ft/s**

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5. Here $h_0 = 0$, so now $h(t) = -16t^2 + v_0 t$ and $v(t) = -32t + v_0$. The initial velocity v_0 and the time T to reach 9 ft in height are unknown. But we can set the height equal to 9 and the velocity equal to 0 to solve for T and v_0.

(a) At time T, $h(T) = 9$ and $v(T) = 0$. Thus,

$-16T^2 + v_0 T = 9$ and $-32T + v_0 = 0 \rightarrow v_0 = 32T$. Now substituting into the first equation, we have $-16T^2 + (32T)T = 9 \rightarrow 16T^2 = 9 \rightarrow T^2 = \frac{9}{16} \rightarrow T = \frac{\sqrt{9}}{4} =$ **0.75 sec**

Then, $v_0 = 32T = 32 \times 0.75 =$ **24 ft/sec**.

(b) Using the time of 0.75 sec and the height of 9 ft of the bouncing ball on Earth, we simply multiply by the ratio of gravities 32/20 to get the time T_R and height H_R of a bouncing ball having the same initial velocity on Rigel:

$$T_R = 0.75 \times \frac{32}{20} = \mathbf{1.2\ sec} \quad \text{and} \quad H_R = 9 \times \frac{32}{20} = \mathbf{14.4\ ft}$$