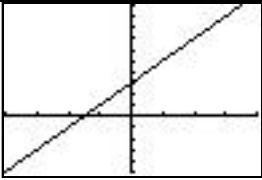
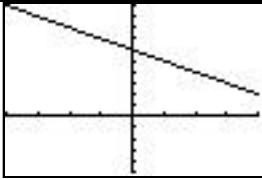
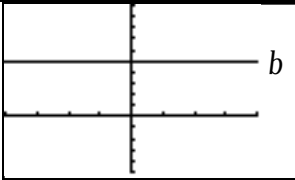


MATH 116**Linear Functions**

A linear function has the form $f(x) = mx + b$, or sometimes written $y = mx + b$. The value m is the *slope*, which determines how quickly the line is increasing or decreasing.

		
$m > 0$ Increasing function	$m < 0$ Decreasing function	$m = 0: y = b$ Constant function

Domain and Range

Unless otherwise restricted, the function $f(x) = mx + b$ is defined for all x . That is, the domain is $-\infty < x < \infty$, which is written as $(-\infty, \infty)$ in interval notation.

When $m \neq 0$, then all possible y -values will be attained by the function. Thus the range is $-\infty < y < \infty$, or $(-\infty, \infty)$ in interval notation. But when $m = 0$, then $f(x) = b$ is a constant function and the range is $y = b$ (which is the only y -value attained).

Evaluating $f(x)$

The function $f(x) = mx + b$ can be evaluated for any specific x value by substituting this value into the formula. For example, consider $f(x) = 3x - 8$. What is the function value for $x = 5$? We evaluate the function at 5 to obtain $f(5) = 3(5) - 8 = 7$.

Intercepts

In the function $f(x) = mx + b$, the value b represents the *y-intercept*, i.e., the y -value where the function crosses y -axis that occurs when $x = 0$. Note that $f(0) = b$.

The *x-intercept* is the value of x for which $y = 0$; that is, it is where the function crosses the x -axis and is found by solving the equation $f(x) = 0$. (Here, set $y = 0$.)

For example, suppose $f(x) = 3x - 8$. The y -intercept is $y = -8$ (which means the point $(0, -8)$ is on the graph of the line). To find the x -intercept, solve for x in the equation $3x - 8 = 0$ to obtain $x = 8/3$. The point $(8/3, 0)$ is also on the graph.

Solving Equations and Inequalities

Given a function $f(x) = mx + b$, we may be asked to solve for the x that results in a specific y -value. For instance, for $f(x) = 3x - 8$, when does $y = 40$? We simply solve the equation $3x - 8 = 40$ as follows: Add 8 $3x = 48$; Divide by 3 $x = 16$. So when $x = 16$, then $y = 40$.

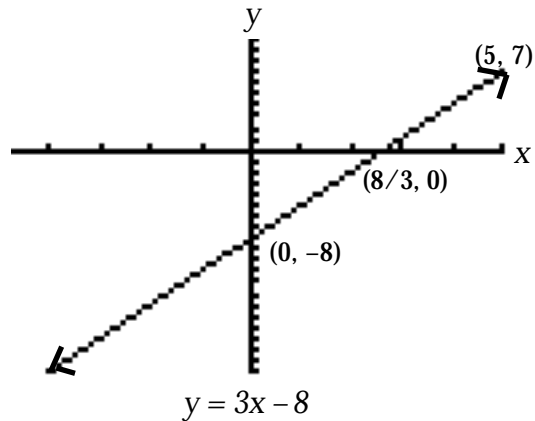
We also may be asked to solve an *inequality*, such as for which x is y more than a specified value c ? Here are the most common inequalities to solve:

y more than c : Solve $mx + b > c$	y less than c : Solve $mx + b < c$	y at least c : Solve $mx + b \geq c$	y at most c : Solve $mx + b \leq c$
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For instance, when is the function $f(x) = 3x - 8$ at least -14 ? Solve $3x - 8 \geq -14$:
 Add 8 $3x \geq -6$ Divide by 3 $x \geq -2$.

Graphing

Given the linear function $f(x) = mx + b$, we can graph the line by plotting two points, generally the x and y -intercepts. If other points on the line have been determined, then we can plot those as well for reference. Here is the graph of $y = 3x - 8$:



Example 1. Let $f(x) = -\frac{7}{4}x + 14$ for $x \leq 10$.

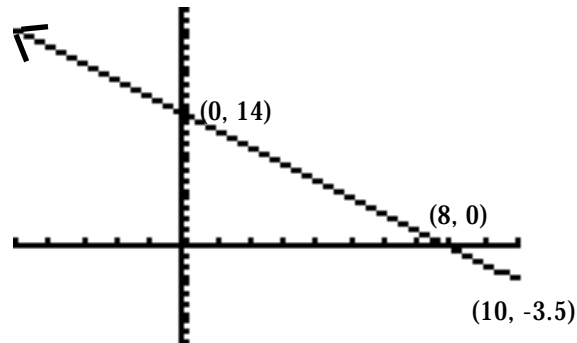
- (a) Find the x and y intercepts.
- (b) Evaluate the function at $x = 10$.
- (c) Graph the function. State the domain and range.
- (d) When will the function equal 35?
- (e) For what values of x will y be at most 21?

Solution. (a) Set $x = 0$: the y -intercept is 14. For the x -intercept, solve $-\frac{7}{4}x + 14 = 0$:

$$-\frac{7}{4}x = -14 \quad \frac{7}{4}x = 14 \quad x = 14 \times \frac{4}{7} = 8 \text{ is the } x\text{-int.}$$

(b) $f(10) = -\frac{7}{4} \times 10 + 14 = -3.5$

(d) Solve $-\frac{7}{4}x + 14 = 35 \quad -\frac{7}{4}x = 21$
 $x = 21 \times \frac{-4}{7} \quad x = -12$



Domain: $(-\infty, 10]$ (allowable x) Range: $[-3.5, \infty)$ (the attained y)

(e) Solve $-\frac{7}{4}x + 14 \leq 21 \quad -\frac{7}{4}x \leq 7 \quad x \geq -4$. Because of domain: $-4 \leq x \leq 10$

Example 2. A car is traveling on the interstate. The amount of gas left in the tank, measured in *gallons*, is a function of the time of travel given by $f(t) = 16 - 2.25t$ for $t \geq 0$, where t is in *hours*.

- (a) What was the initial amount of gas?
 (b) How long until the tank has only half its initial amount? Give the answer precisely in the form of xx hrs, yy min, and zz sec.

Solution. (a) The initial amount of gas at $t = 0$ is $f(0) = \mathbf{16}$ gallons.

(b) Solve $16 - 2.25t = 8$ $8 = 2.25t$ $t = 8/2.25 = 3.555\dots$ hrs.

Use command **3.555... >DMS** (from ANGLE screen) to obtain **3 hrs, 33 min, 20 sec.**

Exercises

1. Let $f(x) = \frac{3}{4}x - 12$ for $x \geq 20$.

- (a) Find the x and y intercepts. (b) Evaluate the function at $x = 8$.
 (c) Graph the function. Label the intercepts and the point found in Part (b).
 State the domain and range.
 (d) When will the function equal -42 ?
 (e) For what values of x will y be at least -9 ?

2. Let $f(x) = -\frac{5}{3}x + 10$.

- (a) Find the x and y intercepts. (b) Evaluate $f(12)$.
 (c) Graph f . Label the intercepts.
 (d) Solve $f(x) = -40$. (e) For what values of x will y be less than 15?

3. A tethered hot air balloon is landing. Its height, measured in *feet*, is a linear function of time t in *seconds* given by $h(t) = 220 - 3.2t$ for $t \geq 0$.

- (a) What was the initial height?
 (b) How long until it touches ground?
 (c) Graph the function for $t \geq 0$. Label the axes appropriately by name, symbol, and unit. State the domain and range.
 (d) For what times will the balloon be at most 100 ft?

4. An orbiting satellite begins to accelerate in order to change its position. Its speed, in *mph*, is a linear function of time t in *hours* given by $s(t) = 17,242 + 60t$ for $t \geq 0$.

- (a) What was the initial speed?
 (b) How long until the speed is 17,386.6 mph? Give the answer precisely in the form of xx hrs, yy min, and zz sec.
 (c) Graph the function and label the axes by name, symbol, and unit.

Solutions

1. (a) Set $x = 0$, then $y = -12$ is y-int

For x-int: Solve $\frac{3}{4}x - 12 = 0$.

Then $x = 12 \times \frac{4}{3} = 16$.

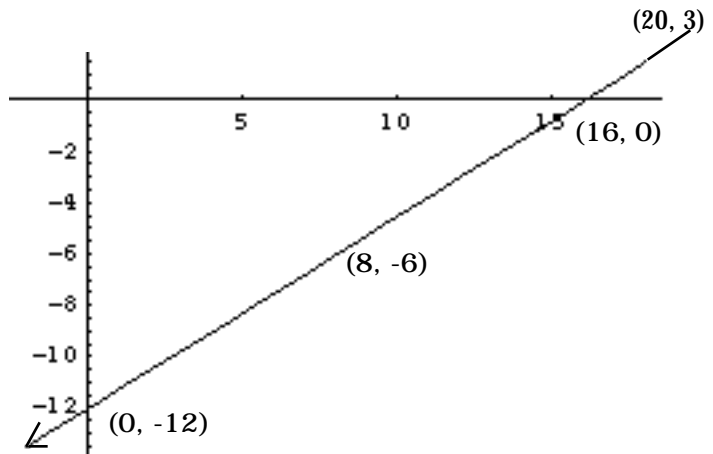
(b) $f(8) = \frac{3}{4}(8) - 12 = -6$.

(d) Solve $\frac{3}{4}x - 12 = -42$ $\frac{3}{4}x = -30$

Then $x = -30 \times \frac{4}{3} = -40$.

(e) Solve $\frac{3}{4}x - 12 \geq -9$ $\frac{3}{4}x \geq 3$

$x \geq 3 \times \frac{4}{3}$ $x \geq 4$ $4 \leq x \leq 20$



(c) Graph of $f(x) = \frac{3}{4}x - 12$

Domain: $x \leq 20$ or $(-\infty, 20]$

Range: $y \geq -3$ or $(-\infty, -3]$.

(Note: $f(20) = 3$)

2. (a) $f(0) = 10$ is y-intercept.

For x-int: Solve $-\frac{5}{3}x + 10 = 0$. Then

$10 = \frac{5}{3}x$ and $x = 30/5 = 6$.

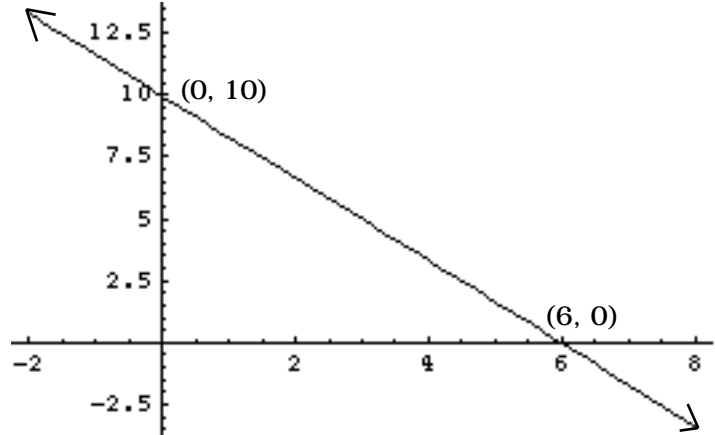
(b) $f(12) = -\frac{5}{3} \times 12 + 10 = -10$.

(d) $-\frac{5}{3}x + 10 = -40$ $50 = \frac{5}{3}x$

$x = 30$.

(e) Solve $-\frac{5}{3}x + 10 < 15$ $-\frac{5}{3}x < 5$

$-5x < 15$ $x > -3$



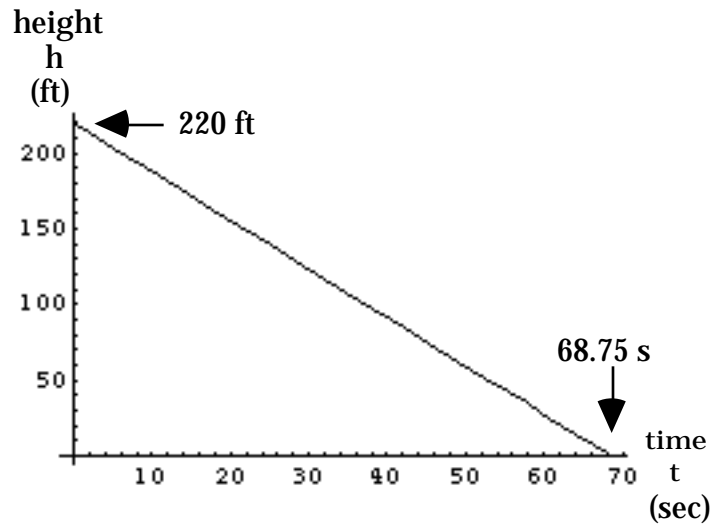
(c) Graph of $f(x) = -\frac{5}{3}x + 10$.

(reverse inequality when dividing by -)

3. (a) At $t = 0$, the balloon started at height $h(0) = 220$ ft.

(b) Solve $220 - 3.2t = 0$. Then the balloon touches down in $t = 220 / 3.2 = 68.75$ sec, or in 1 minute, $8 \frac{3}{4}$ sec.

(c)



Domain: $0 \leq t \leq 68.75$ (sec) Range: $0 \leq h \leq 220$ (ft)

(d) Solve $220 - 3.2t = 100$ $120 = 3.2t$ $120 / 3.2 = t$: For $t = 37.5$ sec, then the balloon is 100 ft or below. (Technically, you may also say for $37.5 \leq t \leq 68.75$ sec.)

4 (a) The initial speed at $t = 0$ is 17,242 mph.

(b) Solve $17,242 + 60t = 17,386.6$ $60t = 144.6$ $t = 144.6 / 60 = 2.41$ hrs.

Use command $2.41 > \text{DMS}$ (from ANGLE screen) to obtain 2 hrs, 24 min, 36 sec.

